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Tactical Extremism*

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Abstract

We provide an instrumental theory of extreme campaign platforms. By adopting an extreme platform, a previously mainstream party with a relatively small probability of winning foregoes such probability altogether. On the other hand, the party builds credibility as the one most capable of delivering an alternative to mainstream policies. The party gambles that if down the road voters become dissatisfied with the status quo and seek something different, the party will be there ready with a credible alternative. In essence, the party sacrifices the most immediate election to invest in greater future success. We call this phenomenon *tactical extremism.* We show under which conditions we expect tactical extremism to arise and we discuss its welfare implications.

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1 Introduction

Consider the challenge of a political party that loses an election, or a sequence of elections, in a two-party system. Party activists must wonder: what should the party change to win the next election? Standard spatial theories (Downs 1957) say the party's platform must have been too far from the median voter's preferred policy, and that to win, the party should moderate its platform, bringing it in line to the median voter's wishes.

However, sometimes a losing party does the opposite: instead of moderating, it doubles down, moving away from moderate policies and embracing radical positions. Spatial models predict that this lurch away from the mainstream and to the extreme will result in subsequent electoral defeats.

For instance, in the United Kingdom, after Labour lost the 2010 and 2015 elections under mainstream candidates (Gordon Brown and Ed Milliband), it elected a far-left figure (Jeremy Corbyn) as its new leader. Labour soon went on to suffer heavy losses of seats in Scottish, Welsh and local council elections in 2016 and 2017 and its electoral prospects became so poor that the UK's Tory government called an early election, anticipating an assured win.[1] Labour similarly followed its 1979 General Election defeat by lurching left toward its greatest electoral defeat in modern times in 1983. In the US, following a defeat in the 1960 election, the GOP chose one of its most extremely conservative senators (Barry Goldwater) as its candidate, leading to a landslide loss in 1964.

We ask why rational, office-motivated parties would choose extremism and near-certain defeat when moderation offers better expectations of victory.

[1] At the election, Labour beat expectations but lost, earning 262 seats, 55 fewer than the Tories.

Non-instrumental theories would account these choices as expressive (Brennan and Hamlin 1998). Extreme policy platforms are more satisfying to purist partisan factions (Roemer 2009, ch. 8), even if they spell electoral doom.

We suggest an alternative, purely instrumental, answer: weak parties go extreme to increase the probability of winning subsequent elections, even at the cost of losing the immediate one.

Consider a two-party system in which parties enjoy policy-specific valence advantages, as in Krasa and Polborn (2010). In particular, assume that the incumbent "owns" the mainstream (Petrocik 1996), in the sense that the voter is more likely to prefer the incumbent to implement the mainstream policy. The opposition party can cede this mainstream ground, investing instead on an a clearly distinguishable alternative policy position, developing credibility and perfecting proposals (Hirsch and Shotts 2005). We call this alternative "extreme" merely because it falls outside the mainstream. By persevering on these position-specific investments, the opposition party thus acquires a policy-specific valence advantage on this alternative policy position. We call this choice by the opposition party "tactical extremism."

As long as the median voter continues to prefer the mainstream policy, tactical extremism results in electoral defeats. However, negative economic outcomes can induce the median voter to update negatively on the virtues of the mainstream policy. If the voter becomes disillusioned, she may wish for an alternative. If so, tactical extremism pays off: the opposition party now enjoys the valence advantage in providing such an alternative. Two assumptions are key: party valence is policy-specific and endogenous; and the voter's future policy preferences are stochastic.

Our analysis shows that a disadvantaged party may choose tactical extremism under the following circumstances:

- confidence in the currently mainstream policy is weak, so that a negative outcome will be interpreted by voters as a policy failure and not bad luck;

- building credibility and competence on a particular policy is particularly important

We also show that a global downturn will further increase the chances of tactical extremism whenever there is relatively little confidence in the mainstream policy but countries with a sufficiently strong consensus or confidence on mainstream policies will *not* experience more tactical extremism after a downturn. A decrease in global volatility will also lead to more tactical extremism if there is relatively little confidence in the mainstream policy and economic prospects are good. Finally, while tactical extremism may seem an unambiguously negative phenomenon, we argue that its welfare effects for voters are ambiguous and actually show how, for some parameters, parties will choose less tactical extremism than voters would like.

In related literature, Calvert (1985) shows that policy-motivated parties may polarize away from the median, and Bernhardt, Duggan and Squintani (2009) show that such polarization may be welfare enhancing. Acemoglu, Egorov and Sonin (2013) argue that parties suspected of conservative leanings choose left-of-center policies to dispel such suspicions. Closest to our work, Aragonés and Palfrey (2004) show that a weak candidate moves away from the center to differentiate itself from a stronger rival. They show that a candidate who could not win at the center, can win with some probability by leaning moderately away from the center; in contrast, we explain why a party that could win at the center chooses instead to lose by lurching to an extreme platform.

2 The Model

Setup. Consider a two-period model of electoral competition with two purely office-motivated parties and one strategic representative voter. In each period $t \in \{1, 2\}$, parties A and D compete in an election. Parties seek to maximize the sum of the probabilities of being elected over the two periods. We consider a policy space $X = \{e, m\}$, where m represents an orthodox, standard policy, and e represents an unorthodox, experimental policy. We refer to m as the *mainstream* policy, and to e as the *extreme* policy.

In each period t, before the election, each party $j \in \{A, D\}$ simultaneously announces a platform $x_t^j \in X$, which corresponds to the policy that the party would implement in period t if it wins office. Let $x_t \equiv (x_t^A, x_t^D)$ and $x^j = (x_1^j, x_2^j)$ and let $x = (x_1; x_2)$. The voter observes x_t and votes for either A or D. The winning party implements its chosen platform.

There are two sources of uncertainty. One is time invariant, and we refer to it as the *state of nature*; the other one is period-specific, and we refer to it as the *economic environment*. An exogenously given time-invariant state of nature $\theta \in \Theta = \{e, m\}$, determines which policy delivers a good economic outcome in a normal economic environment. This state of nature is not known to parties, nor to the voter. We assume that all agents share a common prior that $\Pr[\theta = m] = \mu \in \left(\frac{1}{2}, 1\right)$ and we refer to m as *mainstream* and e as *extreme* precisely because ex-ante, all agents agree that m is more likely to deliver good economic outcomes.

The second exogenous source of uncertainty is the time-variant economic environment.

In each period t, the economic environment is $\omega_t \in \Omega = \{b, n, g\}$, where b is a bad economic environment, n a normal one, and g a good one. Let ω_t be drawn identically and independently in each period from a probability distribution π over $\{b, n, g\}$, where for each $z \in \{b, n, g\}$, π_z represents the probability that $\omega_t = z$.

For each $t \in \{1, 2\}$ we denote with o_t the economic outcome in a given period. We assume that in a normal environment, the outcome is one if the policy matches the state of nature, and zero otherwise; whereas, in a good environment either policy delivers an outcome of one (a rising tide lifts all boats), and in a bad environment neither policy works and they both yield zero. Since $\theta \in \{e, m\}$, we refer to platform/policy θ as the *correct* platform/policy and to the other platform/policy as the *wrong* one.

Voters also care about party attributes. We model *policy-specific* valence (or *competence*) by assuming that whether the government implements its chosen policy competently affects the utility of the voter. We denote with $c_1^j \left(x_1^j\right)$ and $c_2^j \left(x_2^j | x_1^j\right)$ party j's competence in periods 1 and 2 respectively. Competence in the second period is a function of not just the current platform but also of the previous one: acquiring competence on a given policy requires time to build the necessary expertise. We assume that in period 1, $c_1^A (m) = c$ and $c_1^A (e) = 0$, where $c \in \left(0, \frac{1}{4}\right)$ is an exogenous parameter, observed by all players. On the other hand, Party D has no competence ($c_1^D = 0$) on either policy in period 1. Thus, Party A has an exogenous competence advantage on the mainstream platform. The intuition is that both A and D are traditionally mainstream parties with an asymmetry: one of them is perceived as more competent than the other at delivering mainstream policies. We highlight this deliberate asymmetry by henceforth referring to Party A as the *advantaged* party, and

Party D as the *disadvantaged* party. In the second period, for Party A we assume that $c_2^A(m|m) = c_2^A(e|e) = c$ and $c_2^A(e|m) = c_2^A(m|e) = 0$ and for Party D that $c_2^D(e|e) = c$ and $c_2^D(e|m) = c_2^D(m|e) = c_2^D(m|m) = 0$. The interpretation for these assumptions is that Party A owns the mainstream platform, in the sense that it enjoys a policy-specific valence for this platform that D cannot match in two periods but which is nevertheless lost if A ever abandons this platform. On the other hand, the extreme policy position is open and up for grabs in the sense that it is not owned by any party, and a party that commits to it for both periods gains competence on it.

We also model *non-policy* valence (or *charisma*) by assuming that ε_t represents the voter's idiosyncratic preference for Party A in period t. This shock captures non-policy attributes that may nevertheless sway voters in favor of one or the other candidate. For each period $t \in \{1, 2\}$, ε_t is drawn independently from a uniform distribution over $\left[-\frac{1}{4}, \frac{1}{4}\right]$ and its draw is the voter's private information.

Timing. The priors μ on the state of nature and π on the economic environment are common knowledge, but the state θ and the economic environment ω_t for each period t are unknown to all players throughout the game. The non-policy valence ε_t is revealed to the voter at the beginning of period t, but not to the parties. At the start of period 1, Party A is the incumbent party in government, assumed to have implemented m in previous periods. This incumbent can only credibly announce a platform m, so for simplicity we assume that A is bound to announce $x_1^A = m$. party D in opposition, chooses $x_1^D \in \{e, m\}$, and these platforms are publicly observed by all players. Then, the voter chooses in $\{A, D, \emptyset\}$. If the voter chooses a party $j \in \{A, B\}$, then this party wins, while if the voter abstains $(\emptyset)$, the winning party is randomly chosen with equal probability. The winning party $W_1 \in \{A, D\}$ implements policy $x_1^{W_1}$. The economic outcome o_1 is realized and observed by all players.

At the start of period 2, after observing the economic outcome o_1, all players formulate a posterior belief μ^* about which policy is correct. This revision from the prior to the posterior belief may justify a change in the advocated policies, so both parties, including the one in government, are able to formulate a new platform for period 2. Each party $j \in \{A, D\}$ simultaneously chooses $x_2^j \in \{e, m\}$. The platforms profile x_2 is observed by the voter, who then chooses the winning party W_2, which implements its policy, and the economic outcome o_2.

Utilities. Parties are purely office motivated. They maximize the expected number of periods in office.

The voter optimizes period by period, myopically. In each period t, and for each party j, the voter calculates the expected utility that it would attain if she elects party j. This expected utility is computed as the sum of three terms: the expected economic performance under party j (given the voter's beliefs), the policy-specific valence of party j, and the non-policy valence of party j. The voter then optimizes for the period by voting for the party with the highest expected utility.

Solution concept. We assume that parties are strategic and sequentially rational while in each period the voter chooses Party A if the net expected utility function for that period, conditional on her beliefs, is non-negative and Party D otherwise. Also, beliefs follow Bayes' rule and are consistent. We provide a formal definition of belief consistency and of the equilibrium concept in the online Appendix.

We will say that there is *Tactical Extremism* (*TE*) if Party D chooses the extremist platform in the first period. We assume throughout that

$$\mu \geq \frac{1}{2} + \frac{1-4c}{8\pi_n} \equiv \bar{\mu} \tag{1}$$

which guarantees that in the first period, the voter prefers a party with a mainstream policy so that choosing the extremist policy will lead to certain defeat in the first election. This condition simplifies our analysis but also makes it harder for TE to obtain.

3 Analysis

In this section, we informally describe equilibrium behavior and relegate all formal statements and their proofs to the online Appendix.

We start with the policy choice in the second period. If the economic outcome from the first period is good, then both parties will choose the mainstream platform in the second period. If the economic outcome in the first period is bad, it reduces confidence in the mainstream policy. If this confidence in the mainstream policy (μ) was low to begin with, then the posterior belief that the mainstream policy is good is too low after a bad economic outcome, and both parties choose an extremist platform in the second period. If the confidence in the mainstream policy is intermediate, it sinks somewhat but not as much after a bad economic outcome, and Party A sticks to the mainstream platform while Party D proposes the extremist one in the second period. If confidence in the mainstream policy was very high to begin with, the posterior belief that the mainstream policy is correct remains high enough, and both parties propose the mainstream policy in the second period.

Now consider the incentives for Party D to engage in Tactical Extremism (TE), i.e. to propose the extremist policy in the first period, even though at that time, the voter prefers the mainstream policy and will only vote for a party with a mainstream platform. Since by choosing platform e, Party D foregoes any chance of winning the first election, it only has any incentive to propose e in the first period if doing so helps in the second period election. In other words, choosing e in the first period only helps if Party D chooses e again in the second period, and then it enjoys a policy-specific valence advantage c on policy e. As we noted above, in the second period Party D chooses policy e if and only if the prior confidence μ in the mainstream policy is not too high. Therefore, TE only yields any advantage –in any period- if the prior confidence in the mainstream policy is not too high. Furthermore, the magnitude of the advantage is equal to the competence parameter c. This leads us to our main result which we describe more formally in Proposition **1** in the online appendix:

Remark 1 ***Tactical Extremism.*** *The disadvantaged party engages in Tactical Extremism (TE) whenever the voter's initial confidence in the mainstream policy* (μ) *is sufficiently low and competence* (c) *is sufficiently important.*

Competence matters due to two complementary effects. In the first period, if competence matters more, for Party D the probability of winning with the mainstream platform is reduced - its disadvantage is bigger - and this reduces the cost of investing in extremism. The second effect is that in the second period, conditional on a disappointing outcome from the mainstream policy, the investment in expertise on the extremist policy is more valuable and this reinforces the incentives for TE in the first period.

Comparative statics on the effect of the importance of competence (c) or confidence in the mainstream policy (μ) are straightforward: if the importance of competence increases, TE arises for a greater range of prior confidence (μ) in the mainstream policy; and if the confidence μ increases, then TE arises for a smaller range (of very high values) of competence c. In short, the importance of competence necessary for TE to arise in equilibrium increases in the ex-ante confidence in the mainstream policy.

We next consider comparative statics on the effect of changes in the underlying economy over the likelihood of TE. The underlying economy is described by the distribution $\pi = (\pi_b, \pi_n.\pi_g)$ where π_b (resp. π_g) represents the probability that the economic environment is so negative (resp. positive) that a bad (resp. good) economic outcome will occur irrespective of the policy chosen. Conversely, π_n represents the probability that policy matters, and the economic outcome is good if and only if the chosen policy is correct.

Consider the case where the underlying environment becomes unambiguously better, in the sense that the probability of an unconditionally good outcome (π_g) increases, with an equivalent decrease in the probability of an unconditionally bad outcome (π_b). A direct first effect is that a positive economic outcome becomes more likely. This effect reduces the incentives for TE. A second, indirect, effect is that if a bad economic outcome occurs, given that π_b is now lower, the probability that the mainstream policy was to blame is higher, so the posterior confidence on the mainstream policy shrinks faster: a bad outcome is now a stronger negative signal, precisely because it is rarer. This second effect favors TE. If confidence in the mainstream policy (μ) was low to begin with, the strength of the signal doesn't matter: even a weak negative signal sinks the voter's confidence in the mainstream policy, so the first effect dominates. On the other hand, if μ was high, only a strong negative signal induces the voter to prefer the extreme policy to the mainstream one, so the second effect is more important. This logic underlines both of the following results.[2]

Remark 2 ***Impact of Exogenous Factors on Tactical Extremism.*** *If the prior confidence (μ) in the mainstream policy is low, a positive change in the underlying economy reduces the incentives for TE. Whereas, if the prior confidence (μ) in the mainstream policy is high, a positive change in the underlying economy increases the incentives for TE.*

A bit more formally, if μ is low, after an increase in π_g, the minimum value of competence c necessary for TE to occur increases, so the parameter range for which TE occurs shrinks. Whereas, if μ is high, after an increase in π_g, the minimum value of competence c necessary for TE to occur decreases, so the parameter range for which TE occurs expands. The effect of an increase in π_b has exactly the reverse consequences: a worsening of the underlying economy favors TE if there is relatively little trust in the mainstream policy and goes against TE if this trust is high.

We next study the effect on TE of an increase in the probability that policy matters, while π_b and π_g decrease proportionally, so their ratio stays constant. The result and intuition are similar to that in Remark 2 with the difference that the ratio $\frac{\pi_g}{\pi_b+\pi_g}$ now matters as this determines whether an increase in π_n increases or decreases the probability of a good outcome.

Remark 3 ***Impact of Policy Relevance on Tactical Extremism.***

[2] Remark 2 and Remark 3 below respectively draw from Proposition 3 and Proposition 4 in the online appendix.

1. *Suppose the economic environment is favorable* $\left(\frac{\pi_g}{\pi_b+\pi_g}\text{ is high}\right)$. *Then if the initial confidence in the mainstream policy* (μ) *is low, an increase in policy relevance* (π_n) *increases incentives for TE; whereas, if the initial confidence in the mainstream policy* (μ) *is high, an increase in policy relevance* (π_n) *reduces incentives for TE.*

2. *Suppose the economic environment is not favorable* $\left(\frac{\pi_g}{\pi_b+\pi_g}\text{ is low}\right)$. *Then, the effects are reversed.*

3.1 Welfare

Naive intuition would suggest that Tactical Extremism is detrimental to the voter, but there are trade-offs:

In the first period, the effect is unambiguously negative because TE reduces choice since, by Assumption 1, the voter always prefers a party with a mainstream platform. Without TE, the voter has two such parties, and can choose the one with highest aggregate valence (adding up the policy-specific and non-policy valences); with TE, Party D essentially takes itself out of the running for this period. So the voter is worse off.

In the second period, conditional on a bad realization of the first period economic outcome, and conditional on the prior confidence μ on the mainstream policy not being too high, TE is beneficial to the voter. Under these two conditions, the voter has lost confidence in the mainstream policy, and now wants an alternative: the voter wants to elect a party with policy e on its platform. With TE, there is one party (Party D) offering such platform with high competence; without TE, no party would be competent on this extreme policy.

As we detail in Proposition 5, the parameters for which TE is welfare-enhancing do not coincide with the parameters for which TE is an equilibrium phenomenon. We summarize our welfare findings here.

Remark 4 ***Welfare implications.*** *If the initial confidence in the mainstream policy (μ) is low, all equilibrium TE is welfare-enhancing, and there are parameters for which TE is out of equilibrium but it would also be welfare-enhancing (too little TE). On the other hand, the initial confidence in the mainstream policy (μ) is high, if TE does not occur in equilibrium, then it is welfare-reducing, and for some parameters TE occurs in equilibrium but it is welfare-reducing (too much TE).*

If there is sufficient uncertainty about the mainstream policy's effectiveness, the voter would like to "insure" against the risk that the extremist policy turns out to be better, by assuring that one party has competence on it. Party D is more reluctant to go extreme because she fully internalizes that TE brings a loss of the probability of winning the first election, whereas the voter does not care for the identity of the winning party. With strong confidence in the mainstream policy, the voter does not worry so much about the risk described above and focuses on the fact that TE forces her to vote for Party A in the first period.

In sum, TE benefits the voter if she has enough doubts about the mainstream policy that she values having a good alternative ready, in case it might be wanted in the second period.

4 Discussion

We have identified conditions under which one of the two parties in a two-party system adopts an extreme policy for tactical reasons. We predict that a party is more likely to engage in this tactical extremism when:

a) its reputation for policy-specific competence in delivering standard, mainstream policies is poor, and

b) voters' confidence in mainstream policy prescriptions is not too high, and this confidence is at least somewhat likely to dissolve in the future.

With this prediction in mind, we can revisit our motivating example: the Labour Party lurch toward the extreme left in 2015. The financial crisis of 2007-09 and its handling by the New Labour's government under Gordon Brown had battered the reputation for managerial competence of the pro-market, centrist wing of Labour, and at the same time, the crisis and the austerity drives in its aftermath, had undermined voters confidence in mainstream expert policy prescriptions. Conditions for Labour to engage in tactical extremism were more favorable after 2010 than at any other time since the early 1980s, when Labour had last extremism, under Michael Foot. In the 1980s, extremism did not bring success to Labour. We wonder: will it work now?

Our theory would say: "*maybe, but more likely not.*" Tactical extremism is a risky strategy that appeals only to a party at a substantial disadvantage. Adopting an extreme policy, the party condemns itself to an immediate large electoral defeat in hopes of a future electoral gain that may or may not materialize. The party bets against the policy prescriptions that are most likely correct, in hopes that subsequent events will prove these prescriptions wrong after all in the eye of voters. Since these prescriptions are more likely to be right, than to be wrong, tactical extremism is more likely to fail, than to succeed.

References

[1] Acemoglu, Daron, Georgy Egorov, and Konstantin Sonin. "A political theory of populism." *Quarterly Journal of Economics* 128.2 (2013): 771-805.

[2] Aragones, Enriqueta, and Thomas R. Palfrey. "The effect of candidate quality on electoral equilibrium: An experimental study." *American Political Science Review* 98.1 (2004): 77-90.

[3] Bernhardt, Dan, John Duggan, and Francesco Squintani. "The case for responsible parties." *American Political Science Review* 103.4 (2009): 570-587.

[4] Brennan, Geoffrey, and Alan Hamlin. "Expressive voting and electoral equilibrium." *Public choice* 95.1-2 (1998): 149-175.

[5] Calvert, Randall. "Robustness of the Multidimensional Voting Model: Candidate Motivations, Uncertainty, and Convergence." *American Journal of Political Science* 29 (1985): 69–95.

[6] Petrocik, John R. "Issue ownership in presidential elections, with a 1980 case study." *American Journal of Political Science* (1996): 825-850.

[7] Mattias Polborn and Stefan Krasa. "Competition between specialized candidates," *American Political Science Review,* 104.4 (2010): 745 -765.

[8] Roemer, John E. *Political competition: Theory and applications.* Harvard University Press, 2009.

5 Online Appendix

5.1 Formal Notation and Definitions

We denote by $x_t^w \in \{e, m\}$ the policy implemented in period t. Note that for each $j \in \{A, D\}$, $x_t^w = x_t^j$ if the voter votes j in period j, and if the voter abstains, x_t^w is x_t^A or x_t^D with equal probability.

Formally, the economic outcome $o_t : X \times \Theta \times \Omega \longrightarrow \{0, 1\}$ in a given period is a function of the implemented policy, the state of nature, and the economic environment, with the functional form:

$$o_t(x_t^w, \theta, \omega_t) = \begin{cases} 1 \quad if \quad \omega_t = g \text{ or } [\omega_t = n \text{ and } x_t^w = \theta] \\ 0 \quad if \quad \omega_t = b \text{ or } [\omega_t = n \text{ and } x_t^w \neq \theta] \end{cases} .$$

We slightly abuse notation by denoting with $o_t \in \{0, 1\}$ the *realized* economic outcome in period t.

Given our assumption that Party A's platform in the first period is m, a pure strategy $s^A : X \times \{A, D\} \times \{0, 1\} \longrightarrow X$ for Party A is a platform in period 2 as a function of x_1^D, W_1, and o_1. For Party D, a strategy s^D is a pair (s_1^D, s_2^D), where $s_1^D \in X$ is an unconditional choice of platform, and $s_2^D : X \times \{A, D\} \times \{0, 1\} \longrightarrow X$ is a platform in period 2 as a function of x_1^D, W_1, and o_1. For the voter, a strategy s^v is a pair (s_1^v, s_2^v), where $s_1^v : X \times \left[-\frac{1}{4}, \frac{1}{4}\right] \longrightarrow \{A, D\}$ is a party choice in period 1 as a function of x_1^D and the non-policy valence ε_1, and $s_2^v : X \times \{A, D\} \times \{0, 1\} \times X^2 \times \left[-\frac{1}{4}, \frac{1}{4}\right] \longrightarrow \{A, D\}$ is a party choice in period 2 as a function of x_1^D, W_1, o_1, x_2 and ε_2. Let S^A, S^D, and S^v denote the strategy sets of each party and of the voter, respectively.

Party $j's$ optimization problem in period $t = 1$ is:

$$\max_{s^j \in S^j} \left\{ \Pr[W_1 = j|(s^{-j}, s^v)] + \Pr[W_2 = j|(s^{-j}, s^v)] \right\},$$

and in period 2 it reduces to

$$\max_{x_2^j \in \{e,m\}} \Pr[W_2 = j|(s^{-j}, s^v)].$$

The voter's preferences in period t over the two candidates are representable by the following net expected utility function:

$$EU^v(A|x_t) - EU^v(D|x_t) = E[o(x_t^A, \theta, \omega_t)] + c_t^A + \varepsilon_t - E[o(x_t^D, \theta, \omega_t)] - c_t^D,$$

where the first term on the right hand side is the expected economic outcome if Party A is elected, the second and third represent the competence and (relative) charisma of Party A, the fourth is the expected economic outcome if Party D is elected and the fifth is the competence of Party D.

For any implemented policy $x_1^w \in \{m, e\}$ and any economic outcome $o_1 \in \{0, 1\}$, let $\mu^*(x_1^w, o_1) \equiv \Pr[\theta = m|x_1^w, o_1]$ be the posterior probability that the state is m, conditional on observing x_1^w and o_1, and given an unconditional prior probability $\Pr[\theta = m] = \mu$. By Bayes' rule,

$$\mu^*(m, 0) = \frac{\mu \pi_b}{(1-\mu)\pi_n + \pi_b}, \tag{2}$$

and $\mu^*(m,1) = \frac{\mu(\pi_n+\pi_g)}{\mu\pi_n+\pi_g}$, $\mu^*(e,0) = \frac{\mu(\pi_b+\pi_n)}{\pi_b+\mu\pi_n}$, and $\mu^*(e,1) = \frac{\mu\pi_g}{(1-\mu)\pi_n+\pi_g}$.

Definition 1 *Agents' beliefs satisfy consistency if they follow Bayes' rules wherever applicable, and:*

i) period 2 beliefs on θ are equal to μ^*,

ii) beliefs about ε_t and ω_t for each $t \in \{1,2\}$ and at any information set are that ε_t is distributed uniformly in $\left[-\frac{1}{4}, \frac{1}{4}\right]$ and ω_t is distributed according to the probability distribution π.

This consistency requirement means that even off-path, after observing an unexpected action by another player, players stick to their correct beliefs about Nature.[3]

Definition 2 *A strategy profile (s^A, s^D, s^v) and a system of consistent beliefs are an equilibrium if:*

i) Each party $j \in \{A, D\}$ is sequential rational, and, if indifferent at any period t between e or m, it chooses $x_t^j = m$;

ii) In period 1, the voter votes for A if

$$E[o(m,\theta,\omega_t)] + c + \varepsilon_1 \geq E[o(x_1^D,\theta,\omega_1)] \tag{3}$$

and for D otherwise;

iii) In period 2, for any pair of platforms in each period (x_1, x_2), the voter votes for A if

$$E[o(x_2^A,\theta,\omega_t)] + c_2(x_2^A|x_1^A) + \varepsilon_2 \geq E[o(x_2^D,\theta,\omega_1)] + c(x_2^D|x_1^D)$$

[3]A sequential equilibrium satisfies this consistency requirement. A Weak Perfect Bayesian equilibrium need not. A Perfect Bayesian Equilibrium notion is not defined for this game.

and for D otherwise.

5.2 Formal Results and Proofs

We begin by showing that under the assumption that $\mu \geq \overline{\mu}$ the mainstream policy always wins in the first period election.

Lemma 1 *If $x_1 = (m, e)$, then $s_1^v = A$ so $x_1^w = m$ and the probability that Party A wins the election is* 1. *If $x_1 = (m, m)$, then $x_1^w = m$ and the probability that Party A wins the election is $\frac{1}{2} + c$.*

Proof. Recall that the voter is myopic which means that she will maximize only her expected utility from the first period. Then if $x_1 = (m, e)$ the voter expected period payoff from voting A is:

$$E[o(m, \theta, \omega_t)] + c + \varepsilon_1 = \pi_g + \pi_n \mu + c + \varepsilon_1,$$

and the expected period payoff from voting D is

$$E[o(m, \theta, \omega_t)] = \pi_g + \pi_n(1 - \mu),$$

so the net of the two is

$$\begin{aligned} \pi_n \mu + c + \varepsilon_1 - \pi_n(1 - \mu) &= 2\pi_n \mu - \pi_n + c + \varepsilon_1 \\ &\geq 2\pi_n(\frac{1}{2} + \frac{1 - 4c}{8\pi_n}) - \pi_n + c - \frac{1}{4} = 0. \end{aligned}$$

since $\varepsilon_1 \geq -\frac{1}{4}$. This implies that Party A wins with certainty. Conversely, if $E[o(x_1^D, \theta, \omega_1)] = \pi_g + \pi_n \mu$ if $x_1^D = m$ and so Party A wins if $\varepsilon_1 > -c$, which occurs with probability $\frac{1}{2} + 2c$. ■

We next show that in the second period, the platform profile $x_2 = (e, m)$ cannot occur.

Lemma 2 $x_2 = \left(x_2^A, x_2^D\right) = (e, m)$ *is not part of any pure strategy equilibrium.*

Proof. Suppose that $x_2 = (e, m)$ occurred in any pure strategy equilibrium. If the probability of victory for A in this equilibrium is not strictly greater than $\frac{1}{2}$, A deviates to $x_2^A = m$; if it is strictly greater, then D deviates to $x_2^D = e$. ■

The following Lemma shows that if the economic outcome is positive in the first period (and given Lemma 1, this is due to the mainstream policy) then in the second period both parties will choose the mainstream policy.

Lemma 3 *Let* (s^A, s^D, s^v) *be an equilibrium strategy profile. If* $o_1 = 1$, *then* $(s_2^A(x_1^D, s_1^v(x_1^D, \varepsilon_1), 1), s_2^D(x_1^D, s_1^v$
(m, m).

Proof. By Lemma 1, $x_1^w = m$. Given $x_1^w = m$ and $o_1 = 1$, $\mu^*(m, 1) = \mu \frac{\pi_n + \pi_g}{\mu \pi_n + \pi_g} > \mu$ and thus in period 2, if $x_2^j = m$ and $x_2^{-j} = e$, then the voter prefers party j, for any valence realization. Hence, both parties strictly prefer to propose m. ■

We are now ready to study the case where the economic outcome is bad at the end of the first period. We have to consider two cases: that with TE in the first period and that without. We begin with the former. For notational convenience, define the following three cutoffs.

$$\mu_1 \equiv \frac{(\pi_n - c)(1-\pi_g)}{\pi_n(2-2\pi_g-\pi_n-c)},$$
$$\mu_2 \equiv \frac{1-\pi_g}{2(1-\pi_g)-\pi_n},$$
$$\mu_3 \equiv \frac{(\pi_n + c)(1-\pi_g)}{\pi_n(2-2\pi_g-\pi_n+c)}.$$

Lemma 4 *Let (s^A, s^D, s^v) be an equilibrium strategy profile. Then $\left(s_2^A(e,A,0), s_2^D(e,A,0)\right)$ is equal to*

$$\begin{array}{lll} (e,e) & if & \mu \in [0,\mu_1)\,, \\ (m,e) & if & \mu \in [\mu_1,\mu_3)\,, \\ (m,m) & if & \mu \in [\mu_3,1]\,, \end{array}$$

with second period expected utility for Party A

$$\begin{array}{lll} \frac{1}{2}-2c & if & \mu \in [0,\mu_1)\,, \\ \frac{1}{2}-2\pi_n\left(1-2\mu\frac{1-\pi_g-\pi_n}{1-\pi_g-\mu\pi_n}\right) & if & \mu \in [\mu_1,\mu_3)\,, \\ \frac{1}{2}+2c & if & \mu \in [\mu_3,1]\,. \end{array}$$

Proof. Note that $s_1^v(e,\varepsilon_1) = A$ implies $x_1^w = m$, and $x_1^w = m$ and $o_1 = 0$ imply that either that either $\omega_1 = b$ (with probability π_b) or $\omega_1 = n$ and $\theta = e$ (with probability $\pi_n(1-\mu)$), so that

$$\mu^*(m,0) = \mu\frac{1-\pi_g-\pi_n}{1-\pi_g-\mu\pi_n}, \tag{4}$$

which is greater than $\frac{1}{2}$ iff $\mu > \frac{1-\pi_g}{2(1-\pi_g)-\pi_n}$. Let $EU_2^v(j|x_1^D, x_2, \mu^*)$ denote the expected utility for the voter from electing party j, given x_1^D and x_2 and posterior μ^* on the state of nature.

Now since

$$EU_2^v(D|(e,(e,e),\mu^*) = \pi_g + \pi_n(1-\mu^*) + c \text{ and}$$

$$EU_2^v(A|(e,(e,e),\mu^*) = \pi_g + \pi_n(1-\mu^*) + \varepsilon_2$$

then

$$\Pr[A \text{ wins}|(e,(e,e),\mu^*] = \Pr\left[\varepsilon_2 \in \left(c, \frac{1}{4}\right]\right] = \frac{1}{2} - 2c \in \left[0, \frac{1}{2}\right]$$

Whereas,

$$EU_2^v(D|(e,(m,e),\mu^*) = \pi_g + \pi_n(1-\mu^*) + c \text{ and}$$

$$EU_2^v(A|(e,(m,e),\mu^*) = \pi_g + \pi_n\mu^* + c + \varepsilon_2$$

thus

$$\Pr[A \text{ wins}|(e,(m,e),\mu^*(m,0)] = \Pr\left[\varepsilon_2 \in \left(\pi_n(1-2\mu^*(m,0)), \frac{1}{4}\right]\right], \tag{5}$$

which is equal to

$$\begin{aligned} 0 \text{ if } \mu &\in \left(0, \frac{(1-\pi_g)(4\pi_n-1)}{\pi_n(7-8\pi_g-4\pi_n)}\right), \\ \frac{1}{2} - 2\pi_n\left(1-2\mu\frac{1-\pi_g-\pi_n}{1-\pi_g-\mu\pi_n}\right) \text{ if } \mu &\in \left[\frac{(1-\pi_g)(4\pi_n-1)}{\pi_n(7-8\pi_g-4\pi_n)}, \frac{(1-\pi_g)(1+4\pi_n)}{\pi_n(9-8\pi_g-4\pi_n)}\right), \text{ and} \\ 1 \text{ if } \mu &\in \left[\frac{(1+4\pi_n)(1-\pi_g)}{\pi_n(9-8\pi_g-4\pi_n)}, 1\right), \end{aligned}$$

where the cutoffs are obtained by substituting (4) for $\mu^*(m,0)$ in (4) and solving:

$$\pi_n\left(1-2\mu\frac{1-\pi_g-\pi_n}{1-\pi_g-\mu\pi_n}\right)=\mp\frac{1}{4}.$$

Similarly,

$$EU_2^v(D|(e,(m,m),\mu^*)=\pi_g+\pi_n\mu^* \text{ and}$$
$$EU_2^v(A|(e,(m,m),\mu^*)=\pi_g+\pi_n\mu^*+c+\varepsilon_2,$$

so

$$\Pr[A \text{ wins}|(e,(m,m),\mu^*(m,0)]=\Pr[\varepsilon_2\in\left(-c,\frac{1}{4}\right]=\frac{1}{2}+2c\in\left(\frac{1}{2},1\right]$$

Therefore

$$\begin{aligned}\Pr[A \text{ wins}|(e,(e,e),\mu^*(m,0)] &> \Pr[A \text{ wins}|(e,(m,e),\mu^*(m,0)]\Leftrightarrow c<\pi_n\left(1-2\mu^*(m,0)\right)\\ &\Leftrightarrow \mu^*(m,0)<\frac{1}{2}-\frac{c}{2\pi_n}, \text{ and}\\ \Pr[A \text{ wins}|(e,(m,e),\mu^*] &> \Pr[A \text{ wins}|(e,(m,m),\mu^*(m,0)]\Leftrightarrow \pi_n\left(1-2\mu^*(m,0)\right)<-c\\ &\Leftrightarrow \mu^*(m,0)>\frac{1}{2}+\frac{c}{2\pi_n}.\end{aligned}$$

Since A has greater incentives to deviate from (e,e) than D, it follows that $\left(s_2^A(e,A,0),s_2^D(e,A,0)\right)=(e,e)$ is a mutual best response for the second period given $x_1^D=e$, $s_1^v(e,\varepsilon_1)=A$ and $o_1=0$ if and only if $\mu^*(m,0)<\frac{1}{2}-\frac{c}{2\pi_n}$. Similarly, D has greater incentives to deviate from (m,m) than A, so $\left(s_2^A(e,A,0),s_2^D(e,A,0)\right)=(m,m)$ is a mutual best response for the second period given

$x_1^D = e$, $s_1^v(e,\varepsilon_1) = A$ and $o_1 = 0$ if and only if $\mu^*(m,0) > \frac{1}{2} + \frac{c}{2\pi_n}$. If $\mu^* \in \left(\frac{1}{2} - \frac{c}{2\pi_n}, \frac{1}{2} + \frac{c}{2\pi_n}\right)$, then the mutual best response is $\left(s_2^A(e,A,0), s_2^D(e,A,0)\right) = (m,e)$. The intervals for μ follow by simple substitution:

$$\begin{aligned} \mu\frac{1-\pi_g-\pi_n}{1-\pi_g-\mu\pi_n} &= \frac{1}{2} - \frac{c}{2\pi_n} \Leftrightarrow \mu = \frac{(\pi_n - c)(1-\pi_g)}{\pi_n(2-2\pi_g-\pi_n-c)} \text{ and} \\ \mu\frac{1-\pi_g-\pi_n}{1-\pi_g-\mu\pi_n} &= \frac{1}{2} + \frac{c}{2\pi_n} \Leftrightarrow \mu = \frac{(1-\pi_g)(\pi_n + c)}{\pi_n(2-2\pi_g-\pi_n+c)} \end{aligned}$$

■

Next we consider the branch of the tree without TE in the first period.

Lemma 5 *Let (s^A, s^D, s^v) be an equilibrium strategy profile. Then for each $j \in \{A, D\}$, $\left(s_2^A(m,j,0), s_2^D(m,j,0)\right)$ is equal to*

$$\begin{array}{lll} (e,e) & if & \mu \in [0,\mu_1), \\ (m,e) & if & \mu \in [\mu_1,\mu_2), \\ (m,m) & if & \mu \in [\mu_2,1], \end{array}$$

with second period utility for Party A

$$\begin{array}{lll} \frac{1}{2} & if & \mu \in [0,\mu_1), \\ \frac{1}{2} + 2c - 2\pi_n\left(1 - 2\mu\frac{1-\pi_g-\pi_n}{1-\pi_g-\mu\pi_n}\right) & if & \mu \in [\mu_1,\mu_2), \\ \frac{1}{2} + 2c & if & \mu \in [\mu_2,1], \end{array}$$

Proof. Since

$$EU_2^v(D|(m,(e,e),\mu^*) = \pi_g + \pi_n(1-\mu^*) \text{ and}$$
$$EU_2^v(A|(m,(e,e),\mu^*) = \pi_g + \pi_n(1-\mu^*) + \varepsilon_2$$

then

$$\Pr[A \text{ wins}|(m,(e,e),\mu^*] = \Pr\left[\varepsilon_2 \in \left(0, \frac{1}{4}\right]\right] = \frac{1}{2}$$

Whereas,

$$EU_2^v(D|(m,(m,e),\mu^*) = \pi_g + \pi_n(1-\mu^*) \text{ and}$$
$$EU_2^v(A|(m,(m,e),\mu^*) = \pi_g + \pi_n\mu^* + c + \varepsilon_2$$

thus

$$\begin{aligned}\Pr[A \text{ wins}|(e,(m,e),\mu^*(m,0)] &= \Pr\left[\pi_n\mu^*(m,0) + c + \varepsilon_2 > \pi_n(1-\mu^*(m,0))\right] \\ &= \Pr\left[\varepsilon_2 \in \left(\pi_n(1-2\mu^*(m,0)) - c, \frac{1}{4}\right]\right],\end{aligned}$$

which is equal to

$$\begin{aligned} 0 \text{ if } \mu &\in \left[0, \frac{(1-\pi_g)(-1-4c+4\pi_n)}{\pi_n(7-8\pi_g-4\pi_n-4c)}\right) \\ \frac{1}{2} + 2c - 2\pi_n\left(1 - 2\mu\frac{1-\pi_g-\pi_n}{1-\pi_g-\mu\pi_n}\right) \text{ if } \mu &\in \left[\frac{(1-\pi_g)(-1-4c+4\pi_n)}{\pi_n(7-8\pi_g-4\pi_n-4c)}, \frac{(1-\pi_g)(1-4c+4\pi_n)}{\pi_n(9-8\pi_g-4\pi_n-4c)}\right), \text{ and} \\ 1 \text{ if } \mu &\in \left[\frac{(1-\pi_g)(1-4c+4\pi_n)}{\pi_n(9-8\pi_g-4\pi_n-4c)}, 1\right),\end{aligned}$$

where the cutoffs are obtained from

$$\pi_n\left(1-2\mu\frac{1-\pi_g-\pi_n}{1-\pi_g-\mu\pi_n}\right)-c=\mp\frac{1}{4}$$

Similarly,

$$EU_2^v(D|(m,(m,m),\mu^*)=\pi_g+\pi_n\mu^* \text{ and}$$

$$EU_2^v(A|(m,(m,m),\mu^*)=\pi_g+\pi_n\mu^*+c+\varepsilon_2$$

so

$$\Pr[A \text{ wins}|(m,(m,m),\mu^*(m,0)]=\Pr[\varepsilon_2\in\left(-c,\frac{1}{4}\right]=\frac{1}{2}+2c\in\left(\frac{1}{2},1\right].$$

Therefore

$$\begin{aligned}
\Pr[A \text{ wins}|(m,(e,e),\mu^*(m,0)] &> \Pr[A \text{ wins}|(m,(m,e),\mu^*(m,0)]\Leftrightarrow 0<\pi_n(1-2\mu^*(m,0))-c\\
&\Leftrightarrow \mu^*(m,0)<\frac{1}{2}-\frac{c}{2\pi_n}\\
\Pr[A \text{ wins}|(m,(m,e),\mu^*(m,0)] &\geq \Pr[A \text{ wins}|(m,(m,m),\mu^*(m,0)]\Leftrightarrow \pi_n(1-2\mu^*(m,0))-c\leq -c\\
&\Leftrightarrow \mu^*(m,0)\geq\frac{1}{2}.
\end{aligned}$$

Since A has greater incentives to deviate from (e,e) than D, it follows that for each $j\in\{A,D\}$, $\left(s_2^A(m,j,0),s_2^D(m,j,0)\right)=(e,e)$ is a mutual best response for the second period given $x_1^D=m$, $s_1^v(m,\varepsilon_1)=j$ and $o_1=0$ if and only if $\mu_2<\frac{1}{2}-\frac{c}{2\pi_n}$. Similarly, D has greater incentives to deviate from (m,m) than A, so $\left(s_2^A(m,j,0),s_2^D(m,j,0)\right)=(m,m)$ is a mutual best response for the second period given $x_1^D = m$, $s_1^v(m, \varepsilon_1) = j$ and $o_1 = 0$ if and only if $\mu_2 \geq \frac{1}{2}$. If $\mu_2 \in \left(\frac{1}{2} - \frac{c}{2\pi_n}, \frac{1}{2}\right)$, then the mutual best response is $\left(s_2^A(m, j, 0), s_2^D(m, j, 0)\right) = (m, e)$. The intervals for μ follow by simple substitution as before where the new term is

$$\mu \frac{1 - \pi_g - \pi_n}{1 - \pi_g - \mu\pi_n} = \frac{1}{2} \Leftrightarrow \mu = \frac{(1 - \pi_g)}{2(1 - \pi_g) - \pi_n}$$

■

For completeness, we should also consider the actions in the second period, after $x_1^D = e$ and the voter deviates to vote D. Since voters are myopic, this would never occur in equilibrium and so we omit this analysis which is available upon request. We now move to characterizing the expected probability of winning for Party A in the both periods as a function of Party D's decision to pursue TE or not. We then have the following.

Lemma 6 *The total expected utility for Party A over the two periods given $x_1^D = m$ is*

$$\begin{gathered} \tfrac{1}{2} + 2c + \tfrac{1}{2} + 2c\left(\pi_g + \mu\pi_n\right) \text{ if } \mu \in \left(\bar{\mu}, \mu_1\right); \\ \tfrac{1}{2} + 2c + \tfrac{1}{2} + 2c - 2\pi_n\left(1 - \pi_g\right) + 2\mu\pi_n\left(2 - 2\pi_g - \pi_n\right) \text{ if } \mu \in \left[\mu_1, \mu_2\right); \\ \text{and } \tfrac{1}{2} + 2c + \tfrac{1}{2} + 2c \text{ if } \mu \in \left[\mu_2, 1\right). \end{gathered} \tag{6}$$

while the total expected utility for Party A over the two periods given $x_1^D = e$ is

$$\begin{array}{ll} 1 + \frac{1}{2} - 2c\left(1 - 2\pi_g - 2\mu\pi_n\right) & \text{if } \mu \in \left(\bar{\mu}, \mu_1\right); \\ 1 + \frac{1}{2} + 2c\left(\pi_g + \mu\pi_n\right) - 2\pi_n\left(1 - \pi_g\right) + 2\mu\pi_n\left(2 - 2\pi_g - \pi_n\right) & \text{if } \mu \in \left[\mu_1, \mu_3\right); \\ 1 + \frac{1}{2} + 2c & \text{if } \mu \in \left[\mu_3, 1\right). \end{array} \tag{7}$$

Proof. Let $E[P_2(x_1^D)]$ denote the probability that A wins the second period election, as a function of x_1^D, evaluated before the realization of o_1. This can be calculated by noting that in the second period both parties will choose platform profile (m, m) if either $o_1 = 1$ (which happens with probability $\pi_g + \pi_n \mu$) or if $o_1 = 0$ (which happens with probability $\pi_b + \pi_n (1 - \mu)$) and μ is large enough ($\mu \geq \mu_2$ without TE and $\mu \geq \mu_3$ with TE). In the remaining cases, where $o_1 = 0$ and μ is not that large, the probability that A wins follows from substitution from Lemmas 4 and 5. Putting these together with the probabilities of winning in the first period (1 under TE and $\frac{1}{2} + c$ if not) given by 1 provides us the result.

■

We are now ready to describe our main result which builds on the preceding Lemmas. Define $\overline{\mu}_3$ be the value of μ_3 evaluated at $c = \frac{1}{4}$, namely,

$$\overline{\mu}_3 \equiv \frac{(1 - \pi_g)(1 + 4\pi_n)}{\pi_n (9 - 8\pi_g - 4\pi_n)}$$

Proposition 1 *Define the function γ by*

$$\gamma(\mu, \pi_g, \pi_n) = \begin{cases} \frac{1}{4(2 - \pi_g - \mu\pi_n)} & if \quad \mu \in (\bar{\mu}, \mu_2) \\ \frac{1 + 4\pi_n (2\mu - 1 - 2\mu\pi_g + \pi_g - \mu\pi_n)}{4(2 - \pi_g - \mu\pi_n)} & if \quad \mu \in (\mu_2, \overline{\mu}_3] \end{cases}$$

Then, if $\mu \in (\overline{\mu}, \overline{\mu}_3]$ in equilibrium TE occurs if and only if $c \geq \gamma$. Otherwise, there is no TE in equilibrium.

Proof. The probability that Party D wins an election is the reciprocal of the probability that Party A wins an election. Therefore, an equilibrium in which Party D chooses $x_1^D = e$

exists if and only if, for the given μ, the utility value in Expression 7 is strictly lower than the value in Expression 6.

For $\mu \in (\bar{\mu}, \mu_1)$, the condition is

$$\frac{1}{2} + 2c + \frac{1}{2} + 2c(\pi_g + \mu\pi_n) > 1 + \frac{1}{2} - 2c(1 - 2\pi_g - 2\mu\pi_n)$$
$$\Leftrightarrow c > \frac{1}{4(2 - \pi_g - \mu\pi_n)}.$$

For $\mu \in (\mu_1, \mu_2)$, the condition is

$$\frac{1}{2} + 2c + \frac{1}{2} + 2c - 2\pi_n(1 - \pi_g) + 2\mu\pi_n(2 - 2\pi_g - \pi_n) > 1 + \frac{1}{2} + 2c(\pi_g + \mu\pi_n) - 2\pi_n(1 - \pi_g) + 2\mu\pi_n(2 -$$
$$\Leftrightarrow c > \frac{1}{4(2 - \pi_g - \mu\pi_n)}.$$

which the same condition as in the first case.

For $\mu \in (\mu_2, \mu_3)$, the condition is

$$\frac{1}{2} + 2c + \frac{1}{2} + 2c > 1 + \frac{1}{2} + 2c(\pi_g + \mu\pi_n) - 2\pi_n(1 - \pi_g) + 2\mu\pi_n(2 - 2\pi_g - \pi_n)$$
$$\Leftrightarrow c > \frac{1 + 4\pi_n(\mu\pi_n + 2\mu\pi_b - \pi_n - \pi_b)}{4(2 - \pi_g - \mu\pi_n)} = \frac{1 + 4\pi_n(2\mu - 1 - 2\mu\pi_g + \pi_g - \mu\pi_n)}{4(2 - \pi_g - \mu\pi_n)}.$$

We just define γ to be function that represents these lower bounds for different values of μ and it is easy to show that it is continuous. Finally, Since $\frac{\partial \mu_3}{\partial c} = \frac{2}{\pi_n}(\pi_g - 1)\frac{\pi_g + \pi_n - 1}{(c - 2\pi_g - \pi_n + 2)^2} > 0$, the largest value of μ for which this case applies is the case with $\overline{\mu}_3$. ■

We next look at comparative statics, beginning with μ:

Proposition 2 *For any $\mu \in (\bar{\mu}, \overline{\mu}_3)$, γ is a strictly increasing function of μ.*

Proof. For any $\mu \in (\bar{\mu}, \mu_2)$,

$$\frac{\partial}{\partial \mu}\gamma = \frac{1}{4}\frac{\pi_n}{(2-\pi_g-\mu\pi_n)^2} > 0.$$

For any $\mu \in (\mu_2, \overline{\mu}_3)$,

$$\begin{aligned}\frac{\partial}{\partial \mu}\gamma &= \frac{d}{d\mu}\left(\frac{1+4\pi_n(2\mu-1-2\mu\pi_g+\pi_g-\mu\pi_n)}{4(2-\pi_g-\mu\pi_n)}\right)\\ &= \frac{1}{4}\pi_n\frac{-24\pi_g-12\pi_n+8\pi_g^2+8\pi_g\pi_n+17}{(\pi_g+\mu\pi_n-2)^2},\end{aligned}$$

which is strictly positive if and only if

$$-24\pi_g-12\pi_n+8\pi_g^2+8\pi_g\pi_n+17>0,$$

which holds. Finally, γ is not differentiable at $\mu=\mu_2$ but since $\frac{\partial}{\partial\mu}\gamma\Big|_{\mu\to(\mu_2)^+}$ and $\frac{\partial}{\partial\mu}\gamma\Big|_{\mu\to(\mu_2)^-}$ are strictly positive, the result still holds. ■

The Proposition below looks at the comparative statics when we increase π_g for constant π_n. Define

$$\tilde{\pi}_g = \left(\frac{1}{4}\sqrt{2}\right)\frac{2\pi_n+2\sqrt{2}\sqrt{2\pi_n^2+1}+2\pi_n^2-\sqrt{2}\pi_n\sqrt{2\pi_n^2+1}-4}{\sqrt{2\pi_n^2+1}-\sqrt{2}} \text{ and}$$

$$\widetilde{\mu} = \frac{1}{2}+\frac{1}{2\pi_n}-\frac{\sqrt{4\pi_n^2+2}}{4\pi_n}.$$

Proposition 3 *If $\pi_g \leq \pi_g^*$, then γ is a strictly increasing function of π_g for $\mu < \mu_2$ and strictly decreasing for $\mu \in (\mu_2, \overline{\mu}_3)$. If $\pi_g > \pi_g^*$ then $\widetilde{\mu} > \mu_2$ and γ is a strictly increasing function of π_g for $\mu < \widetilde{\mu}$ and strictly decreasing for $\mu > \widetilde{\mu}$.*

Proof. For any $\mu \in (\bar{\mu}, \mu_2)$,

$$\frac{\partial}{\partial \pi_g}\gamma = \frac{1}{4\left(2 - \pi_g - \mu\pi_n\right)^2} > 0.$$

For any $\mu \in (\mu_2, \overline{\mu}_3)$,

$$\frac{\partial}{\partial \pi_g}\gamma = \frac{1 + 4\pi_n(-2\mu + 2\mu^2\pi_n + 1 - 2\mu\pi_n)}{4(2 - \pi_g - \mu\pi_n)^2}$$

which is strictly positive if

$$1 + 4\pi_n(-2\mu + 2\mu^2\pi_n + 1 - 2\mu\pi_n) > 0$$

$$\Leftrightarrow \mu < \widetilde{\mu},$$

and strictly negative if vice-versa.

Note that for $\pi_g \leq 1 - \pi_n$ (true by definition) and $\pi_n \geq \frac{1}{4}$ (true by assumption), $\pi_g \leq \pi_g^*$ implies $\widetilde{\mu} \leq \mu_2$, and then $\mu \in (\mu_2, \overline{\mu}_3)$ implies $\mu > \widetilde{\mu}$, so if $\pi_g \leq \pi_g^*$, γ is a strictly increasing function of π_g for $\mu < \mu_2$ and strictly decreasing for $\mu \in (\mu_2, \overline{\mu}_3)$. Whereas, if $\pi_g > \pi_g^*$, then $\widetilde{\mu} > \mu_2$ and $\frac{\partial}{\partial \pi_g}\gamma$ is strictly positive for $\mu < \widetilde{\mu}$ and strictly negative for $\mu > \widetilde{\mu}$. ■

Now instead we increase π_n leaving the ratio $\rho = \dfrac{\pi_b}{\pi_g + \pi_b}$ constant.

Proposition 4 *There exists values*

$$\pi_b^* = \frac{(1-\rho)(2-\mu)}{\rho-\mu} - \frac{1}{2}(1-\rho)\frac{\sqrt{(\mu+\rho-2\mu\rho)(24\mu+8\rho-17\mu^2-5\rho^2-8\mu^2\rho^2-42\mu\rho+16\mu\rho^2+24\mu^2\rho)}}{(\rho-\mu)(\mu+\rho-2\mu\rho)} \text{ and}$$

$$\mu_\rho = \frac{-11\rho+4\rho^2+8}{-24\rho+8\rho^2+17}$$

such that if $\pi_b \geq \pi_b^*$ and $\mu > \mu_\rho$ then γ is a strictly increasing function of π_n whereas if either of those conditions are strictly not satisfied then γ is a strictly decreasing function of π_n.

Proof. We have two cases

1. For any $\mu \in \left(\bar{\mu}, \frac{1-\pi_g}{2-2\pi_g-\pi_n}\right) = (\bar{\mu}, \mu_2)$,

   $$\gamma = \frac{1-\rho}{4(-\mu-2\rho+\mu\rho+\mu\pi_b-\rho\pi_b+2)}$$

   so that the derivative of this with respect to π_b, given the constraint, is equivalent to an increase in $\pi_g + \pi_b$ and so a decrease in π_n. Thus,

   $$\frac{\partial\gamma}{\partial\pi_b} > 0 \Leftrightarrow \frac{\partial\gamma}{\partial\pi_n} < 0$$

   So

   $$\frac{\partial\gamma}{\partial\pi_b} = \frac{1}{4}(1-\rho)\frac{\rho-\mu}{(-\mu-2\rho+\pi_b\mu-\pi_b\rho+\mu\rho+2)^2}$$

   This implies

   $$\frac{\partial\gamma}{\partial\pi_n} > 0 \Leftrightarrow \mu > \rho$$

2. $\mu \in \left(\mu_2, \frac{(1-\pi_g)(1+4\pi_n)}{\pi_n(9-8\pi_g-4\pi_n)}\right)$. Now

$$\gamma = \frac{1}{4(1-\rho)} \frac{4\mu+6\rho+4\pi_b-3\rho^2-8\mu\rho+4\mu\rho^2-4\mu\pi_b^2-4\rho\pi_b^2-4\rho^2\pi_b+8\mu\rho\pi_b^2+8\mu\rho^2\pi_b-8\mu\rho\pi_b-3}{-\mu-2\rho+\mu\rho+\mu\pi_b-\rho\pi_b+2}$$

and

$$\frac{\partial\gamma}{\partial\pi_b} = \frac{1}{4} \frac{4(\rho-\mu)(\mu+\rho-2\mu\rho)\pi_b^2-8(1-\rho)(2-\mu)(\mu+\rho-2\mu\rho)\pi_b+(1-\rho)^2\left(-\mu+5\rho-4\mu^2-16\mu\rho+8\mu^2\rho+8\right)}{(1-\rho)(-\mu-2\rho+\pi_b\mu-\pi_b\rho+\mu\rho+2)^2}$$

The sign depend on the sign of the numerator which is a quadratic function of π_b with extremum at

$$\pi_b^* = \frac{(1-\rho)(2-\mu)}{\rho-\mu}$$

and two roots

$$\begin{aligned}
\pi_b^A &= \frac{(1-\rho)(2-\mu)}{\rho-\mu} + \frac{1}{2}(1-\rho)\frac{\sqrt{(\mu+\rho-2\mu\rho)(24\mu+8\rho-17\mu^2-5\rho^2-8\mu^2\rho^2-42\mu\rho+16\mu\rho^2+24\mu^2\rho)}}{(\rho-\mu)(\mu+\rho-2\mu\rho)} \\
\pi_b^B &= \frac{(1-\rho)(2-\mu)}{\rho-\mu} - \frac{1}{2}(1-\rho)\frac{\sqrt{(\mu+\rho-2\mu\rho)(24\mu+8\rho-17\mu^2-5\rho^2-8\mu^2\rho^2-42\mu\rho+16\mu\rho^2+24\mu^2\rho)}}{(\rho-\mu)(\mu+\rho-2\mu\rho)}
\end{aligned}$$

It can be shown that for our parameter values both roots are real valued. This means that if $\rho > \mu$ then we have a strictly convex quadratic function with global minimum at π_b^* which has roots at $\pi_b^A > \pi_b^*$ and $\pi_b^B < \pi_b^*$. So this is negative in the interval $\left(\pi_b^B, \pi_b^A\right)$. If $\rho < \mu$ then we have a strictly concave function with global maximum at π_b^* which has roots at $\pi_b^A < \pi_b^*$ and $\pi_b^B > \pi_b^*$. So this is positive in the interval $\left(\pi_b^A, \pi_b^B\right)$.

Further, since

$$\pi_g + \pi_b = \frac{1}{1-\rho}\pi_b = 1 - \pi_n \Rightarrow \frac{1}{1-\rho}\pi_b \leq 1 \Leftrightarrow \pi_b \leq 1 - \rho$$

it easy to see that if $\rho > \mu$ then $\pi_b^* > 1-\rho$ while if $\rho < \mu$ then $\pi_b^* < 0$. So:

- If $\rho > \mu$ then we have a strictly convex function with constrained global minimum at $1-\rho$. So this is negative in the interval $\left(\min(\pi_b^B, 1-\rho), 1-\rho\right)$
- If $\rho < \mu$ then we have a strictly concave function with constrained global maximum at 0. So this is positive in the interval $\left(0, \max\left(\pi_b^B, 0\right)\right)$

From now on, let $\pi_b^* = \pi_b^B$. We can now compare π_b^* with 0 and $1-\rho$. We can show that

$$\tfrac{(1-\rho)(2-\mu)}{\rho-\mu} - \frac{1}{2}(1-\rho)\,\tfrac{\sqrt{(\mu+\rho-2\mu\rho)(24\mu+8\rho-17\mu^2-5\rho^2-8\mu^2\rho^2-42\mu\rho+16\mu\rho^2+24\mu^2\rho)}}{(\rho-\mu)(\mu+\rho-2\mu\rho)} > 0$$

iff $\mu < \frac{1}{16\rho-8}\left(16\rho - \sqrt{3}\sqrt{-48\rho+32\rho^2+43}+1\right)$ or $\mu > \frac{1}{16\rho-8}\left(16\rho + \sqrt{3}\sqrt{-48\rho+32\rho^2+43}+1\right)$

and since both expressions are greater than one this always holds. So $\pi_b^* > 0$. Now

$$\tfrac{(1-\rho)(2-\mu)}{\rho-\mu} - \frac{1}{2}(1-\rho)\,\tfrac{\sqrt{(\mu+\rho-2\mu\rho)(24\mu+8\rho-17\mu^2-5\rho^2-8\mu^2\rho^2-42\mu\rho+16\mu\rho^2+24\mu^2\rho)}}{(\rho-\mu)(\mu+\rho-2\mu\rho)} < 1-\rho$$

$$\text{iff } \mu > \tfrac{-11\rho+4\rho^2+8}{-24\rho+8\rho^2+17} = \mu_\rho$$

Noting that $\mu_\rho > \rho$ iff $\rho < \frac{1}{2} < \mu$ we can summarize this as follows:

a. If $\mu \leq \mu_\rho < \rho$ then the derivative of interest (with respect to π_b) is positive.

b. If $\rho > \mu > \mu_\rho$ then the derivative of interest is positive in the interval $(0, \pi_b^*)$ and negative otherwise.

c. If $\mu > \mu_\rho > \rho$ then the derivative of interest is positive in the interval $\left(0, \pi_b^B\right)$ and negative otherwise.

but this implies that

$$\frac{\partial \gamma}{\partial \pi_n} > 0 \Leftrightarrow \mu > \mu_\rho \text{ and } \pi_b \geq \pi_b^*$$

■

We should add that μ_ρ is increasing in ρ. This means that for a given π_n, the condition $\frac{\partial \gamma}{\partial \pi_n} > 0$ tends to hold when π_b is high relative to π_g and the reverse otherwise. We now turn to welfare analysis where the comparison is between the equilibrium outcome in Proposition 1 and the platform choices that a social planner would choose if this planner is trying to maximize the voter's utility over two periods. We obtain the result below which implies that in equilibrium there is too little TE from the planner's perspective when μ is relatively low and too much when μ is relatively high.

Proposition 5 *There exist a value $\widehat{\mu} \in (\mu_2, \mu_3)$ and $\gamma^O \in \left[0, \frac{1}{4}\right]$ such that the social planner prefers tactical extremism iff $c > \gamma^O$. If $\mu < \widehat{\mu}$ then $\gamma^O < \gamma$ whereas if $\mu > \widehat{\mu}$ then $\gamma^O > \gamma$.*

Proof. We consider three cases where $\overline{\mu} < \mu < \mu_1$, $\mu_1 \leq \mu < \mu_2$ and $\mu_2 \leq \mu < \mu_3$ and for each we determine conditions on c such that the planner would prefer TE and then compare with the equilibrium γ. Note first that in the first election, if both parties choose platform m then the voter has a choice of voting for A which gives her non-policy utility

$c+\varepsilon_1$ or for Party D which gives her non-policy utility 0. In case of TE, given our parametric assumptions, then A will win for sure so that the voter gets non-policy utility $c+\varepsilon_1$. This means that TE lowers the voter's expected utility whenever $c+\varepsilon_1 < 0$ or, more precisely, the expected loss from TE is

$$E\left[c+\varepsilon_1 | c+\varepsilon_1 < 0\right] \Pr[c+\varepsilon_1 < 0] = \int_{-\frac{1}{4}}^{-c} 2\left(c+t\right) dt = -c^2 + \frac{1}{2}c - \frac{1}{16}.$$

1. If $\mu \in (\overline{\mu}, \mu_1)$, then we have that conditional on a bad outcome, TE has an advantage since $\max\left(\varepsilon_2, c\right) > \max\left(\varepsilon_2, 0\right)$. In particular, this matters when $\varepsilon_2 < c$ and so the expected gain from TE is

$$\begin{aligned} & E[c-\varepsilon_2 | \varepsilon_2 \in (0,c)] \Pr[\varepsilon_2 \in (0,c)] + c \Pr[\varepsilon_2 < 0] \\ = & \int_0^c 2(c-t)dt + c\int_{-\frac{1}{4}}^{0} 2dt = \frac{1}{2}c\left(2c+1\right). \end{aligned}$$

The voter prefers TE if

$$\begin{aligned} -c^2 + \frac{1}{2}c - \frac{1}{16} + \left(\pi_b + (1-\mu)\,\pi_n\right)\frac{1}{2}c\left(2c+1\right) > 0 \\ \Leftrightarrow -\left(\pi_g + \mu\pi_n\right)c^2 + \frac{1}{2}\left(2-\pi_g-\mu\pi_n\right)c - \frac{1}{16} > 0 \\ \Leftrightarrow -\Lambda c^2 + \frac{1}{2}\left(2-\Lambda\right)c - \frac{1}{16} > 0, \end{aligned}$$

where we have used the substitutions $\pi_b = 1-\pi_g-\pi_n$ and $\Lambda = \pi_g + \mu\pi_n$. Then the two roots are

$$\frac{2+\sqrt{-5\Lambda+\Lambda^2+4}}{4\Lambda}-\frac{1}{4} \text{ and } \frac{2-\sqrt{-5\Lambda+\Lambda^2+4}}{4\Lambda}-\frac{1}{4},$$

The term under square root is positive and so the two roots are well defined. Also, we can rewrite the two roots as

$$\frac{1}{2\Lambda}-\frac{1}{4}+f \text{ and } \frac{1}{2\Lambda}-\frac{1}{4}-f,$$

where $f=\frac{\sqrt{-5\Lambda+\Lambda^2+4}}{4\Lambda}>0$ and since $\Lambda<1$ the first root is greater than $\frac{1}{4}$, which means it is outside of the admissible range. So, the correct root is $\gamma^O=\frac{1}{2\Lambda}-\frac{1}{4}-f$. Comparing γ^O to the equilibrium boundary

$$\gamma=\frac{1}{4\left(2-\pi_g-\mu\pi_n\right)}=\frac{1}{4\left(2-\Lambda\right)}$$

yields

$$\frac{1}{2\Lambda}-\frac{1}{4}-f<\frac{1}{4\left(2-\Lambda\right)}$$

so for $\mu<\mu_1$, $\gamma^O<\gamma$.

2. If $\mu\in(\mu_1,\mu_2)$ then we have that conditional on a bad outcome, TE yields a second period benefit, since $\max\{\mu^*(m,0)+c+\varepsilon_2, 1-\mu^*(m,0)+c\}>\max\{\mu^*(m,0)+c+$

$\varepsilon_2, 1-\mu^*(m,0)\}$. In particular, there is no benefit from TE if

$$\mu^*(m,0)+c+\varepsilon_2 > 1-\mu^*(m,0)+c,$$

or equivalenlty, if $\varepsilon_2 > 1-2\mu^*(m,0)$; whereas if

$$1-\mu^*(m,0)+c > \mu^*(m,0)+c+\varepsilon_2 > 1-\mu^*(m,0),$$

or equivalently, if $\varepsilon_2 \in (1-2\mu^*(m,0)-c, 1-2\mu^*(m,0))$, then the expected gain from TE (times the probability for this case) is

$$\int_{1-2\mu^*(m,0)-c}^{1-2\mu^*(m,0)} 2\left(1-\mu^*(m,0)+c-\mu^*(m,0)-c-t\right)dt = c^2.$$

Finally, if

$$1-\mu^*(m,0)+c > 1-\mu^*(m,0) > \mu^*(m,0)+c+\varepsilon_2,$$

or equivalently, $\varepsilon_2 < 1-2\mu^*(m,0)-c$, then the expected gain from TE (times the probability for this case) is

$$\int_{-\frac{1}{4}}^{1-2\mu^*(m,0)-c} 2c dt = \frac{1}{2}c\left(5-4c-8\mu^*(m,0)\right).$$

This means that the voter prefers TE whenever

$$-c^2+\frac{1}{2}c-\frac{1}{16}+\left(\pi_b+(1-\mu)\pi_n\right)\left(\frac{1}{2}c\left(5-4c-8\mu^*(m,0)\right)+c^2\right) > 0. \tag{8}$$

Substituting in the value of $\mu^*(m,0)$ (Expression 2), inequality 8 becomes

$$-\left(2-\pi_g-\mu\pi_n\right)c^2+\frac{1}{2}\left(1+\left(\pi_b+\left(1-\mu\right)\pi_n\right)\left(5-8\frac{\mu\pi_b}{(1-\mu)\pi_n+\pi_b}\right)\right)c-\frac{1}{16}>0$$
$$\Leftrightarrow-\left(2-\pi_g-\mu\pi_n\right)c^2+\frac{1}{2}\left(-8\mu-5\pi_g+8\mu\pi_g+3\mu\pi_n+6\right)c-\frac{1}{16}>0$$
$$\Leftrightarrow-\Lambda c^2+\frac{1}{2}\left(2-\Lambda\right)c-\frac{1}{16}+2\left[\left(\Lambda-1\right)c^2+\left(\left(2\mu-1\right)\Lambda+1-2\mu\left(-\pi_n+\mu\pi_n+1\right)\right)c\right]>0$$
$$\Leftrightarrow F\left(c,\mu\right)=-\Lambda c^2+\frac{1}{2}\left(2-\Lambda\right)c-\frac{1}{16}+2\left[\left(\Lambda-1\right)c^2+\left(\left(2\mu-1\right)\left(\pi_g-1\right)+\mu\pi_n\right)c\right]>0,$$

where $-\Lambda c^2+\frac{1}{2}\left(2-\Lambda\right)c-\frac{1}{16}$ is the condition from the previous case. Consider the term in square brackets. Its derivative with respect to μ is negative:

$$\frac{d\left(\left[\left(\Lambda-1\right)c^2+\left(\left(2\mu-1\right)\left(\pi_g-1\right)+\mu\pi_n\right)\right]_{\Lambda=\pi_g+\mu\pi_n}\right)}{d\mu}=2\pi_g+\pi_n+c^2\pi_n-2<0$$

for any c. The derivative of the term outside square brackets is also negative

$$\frac{d\left(\left[-\Lambda c^2+\frac{1}{2}\left(2-\Lambda\right)c-\frac{1}{16}\right]_{\Lambda=\pi_g+\mu\pi_n}\right)}{d\mu}=-\frac{1}{2}c\pi_n\left(2c+1\right)$$

for any c. So the expression $F\left(c,\mu\right)$ as a function of μ, for any c is minimized in our range for $\mu=\mu_2$. Now, this is a function $F\left(c,\mu\right)$ quadratic and concave in c which is negative for $c=0$ and so if we select the smallest possible μ the roots in c that solve

$$F\left(c,\mu\right)=0$$

are going to be the closest to the maximum. If the upper root is above $c=\frac{1}{4}$ then only the lower root matters and this will therefore be the highest possible value of the root in our range - the worst case scenario. We now study these roots. So

$$F(c,\mu_2)=\frac{1}{16}\left(16c+16c^2\Lambda-8c\Lambda-32c^2-1\right)$$

which has roots

$$\frac{1}{4}\frac{2-\Lambda+\sqrt{-3\Lambda+\Lambda^2+2}}{2-\Lambda}\text{ and }\frac{1}{4}\frac{2-\Lambda-\sqrt{-3\Lambda+\Lambda^2+2}}{2-\Lambda}$$

The term in square roots is always positive, so the first root above is clearly greater than $\frac{1}{4}$. So the relevant root is

$$\frac{1}{4}\frac{2-\Lambda-\sqrt{-3\Lambda+\Lambda^2+2}}{2-\Lambda}$$

and it is easy to see that $\frac{1}{4}\frac{2-\Lambda-\sqrt{-3\Lambda+\Lambda^2+2}}{2-\Lambda}<\gamma$. This is the worst-case scenario cut-off so, although this is not γ^O, still we must have $\gamma^O\leq\frac{1}{4}\frac{2-\Lambda-\sqrt{-3\Lambda+\Lambda^2+2}}{2-\Lambda}<\gamma$.

3. If $\mu_2\leq\mu<\mu_3$ then we have that conditional on a bad outcome, TE does not have an obvious advantage. We have

$$1-\mu^*+c>\mu^*\Leftrightarrow\mu^*<\frac{1+c}{2}$$
$$\Leftrightarrow\mu<\frac{(1-\pi_g)(1+c)}{2-2\pi_g-\pi_n(1-c)}$$

but

$$
\begin{aligned}
\mu_3 - \frac{(1-\pi_g)(1+c)}{2-2\pi_g-\pi_n(1-c)} &= 2c(1-\pi_n)(1-\pi_g)\frac{1-\pi_g-\pi_n}{\pi_n(2-2\pi_g-\pi_n+c)(2-2\pi_g-\pi_n(1-c))} > 0 \\
\frac{(1-\pi_g)(1+c)}{2-2\pi_g-\pi_n(1-c)} - \mu_2 &= 2c(1-\pi_g)\frac{1-\pi_g-\pi_n}{(2-2\pi_g-\pi_n)(2-2\pi_g-\pi_n(1-c))} > 0
\end{aligned}
$$

so that this condition discriminates between the two cases. This means that if

$$\frac{(1-\pi_g)(1+c)}{2-2\pi_g-\pi_n(1-c)} < \mu < \mu_3$$

then there is either no advantage of TE or a disadvantage. We therefore, from now on, assume

$$\mu_2 < \mu < \frac{(1-\pi_g)(1+c)}{2-2\pi_g-\pi_n(1-c)}$$

In that case, if

$$\mu^* + c + \varepsilon_2 > 1 - \mu^* + c > \mu_2 \Leftrightarrow \varepsilon_2 > 1 - 2\mu^*$$

then there is no advantage to TE. If

$$1 - \mu^* + c > \mu^* + c + \varepsilon_2 > \mu^* \Leftrightarrow -c < \varepsilon_2 < 1 - 2\mu^*$$

then the expected gain from TE (times the probability for this case) is

$$\int_{-c}^{1-2\mu^*} 2(1-\mu^*+c-\mu^*-c-t)\,dt = (c-2\mu^*+1)^2$$

If

$$1-\mu^*+c>\mu^*>\mu^*+c+\varepsilon_2 \Leftrightarrow -c>\varepsilon_2$$

then the expected gain from TE (times the probability for this case) is

$$\int_{-\frac{1}{4}}^{-c} 2\left(1-\mu^*+c-\mu^*\right)dt=\frac{1}{2}\left(1-4c\right)\left(c-2\mu^*+1\right)$$

This means that the voter prefers TE whenever

$$-c^2+\frac{1}{2}c-\frac{1}{16}+\left(\pi_b+\left(1-\mu\right)\pi_n\right)\left(\left(c-2\mu^*+1\right)^2+\frac{1}{2}\left(1-4c\right)\left(c-2\mu^*+1\right)\right)>0$$
$$\Leftrightarrow -\left(2-\pi_g-\mu\pi_n\right)c^2+\frac{1}{2}\left(2-\pi_g-\mu\pi_n\right)c-\frac{1}{16}\Sigma>0$$
$$\Leftrightarrow -\left(2-\Lambda\right)c^2+\frac{1}{2}\left(2-\Lambda\right)c-\frac{1}{16}\Sigma=G\left(c,\mu\right)>0$$

where

$$\Sigma=\frac{80\mu+47\pi_g-64\mu^2-24\pi_g^2-64\mu^2\pi_g^2-8\mu^2\pi_n^2-160\mu\pi_g-33\mu\pi_n+80\mu\pi_g^2+128\mu^2\pi_g+48\mu^2\pi_n-48\mu^2\pi_g\pi_n+32\mu\pi_g\pi_n-23}{1-\Lambda}$$

Now, $G\left(c,\mu\right)$ has two roots

$$\frac{1}{4}\frac{2-\Lambda-\sqrt{\left(2-\Lambda\right)\left(2-\Sigma-\Lambda\right)}}{2-\Lambda}\text{ and }\frac{1}{4}\frac{2-\Lambda+\sqrt{\left(2-\Lambda\right)\left(2-\Sigma-\Lambda\right)}}{2-\Lambda}$$

where the second root for the usual arguments does not apply. In the first root the term in square root will be positive for $\mu = \mu_2$ because

$$\left[\frac{1}{4}\frac{2-\Lambda-\sqrt{(2-\Lambda)(2-\Sigma-\Lambda)}}{2-\Lambda}\right]_{\mu=\mu_2} = \left[\frac{1}{4}\frac{2-\Lambda-\sqrt{-3\Lambda+\Lambda^2+2}}{2-\Lambda}\right]_{\mu=\mu_2}$$

where the term on the l.h.s. is the root we studied in the previous case. We know this is well-defined and implies

$$G(c,\mu_2) = F(c,\mu_2)$$

so let

$$\gamma^O = \frac{1}{4}\frac{2-\Lambda-\sqrt{(2-\Lambda)(2-\Sigma-\Lambda)}}{2-\Lambda}$$

Now note also that

$$G\left(c, \frac{(1-\pi_g)(1+c)}{2-2\pi_g-\pi_n(1-c)}\right) = -\frac{1}{16}(4c-1)^2 < 0$$

All of this implies that if γ^O is a strictly increasing function of μ (clearly it is continuous), there must exist a $\widehat{\mu} < \frac{(1-\pi_g)(1+c)}{2-2\pi_g-\pi_n(1-c)}$ such that

$$\gamma^O(\widehat{\mu}) = \gamma(\widehat{\mu}) = \frac{1+4\pi_n(\widehat{\mu}\pi_n+2\widehat{\mu}\pi_b-\pi_n-\pi_b)}{4(2-\pi_g-\widehat{\mu}\pi_n)}$$

and $\gamma^O < \gamma$ for $\mu < \widehat{\mu}$ and vice-versa for $\mu > \widehat{\mu}$ proving our result. So now we study

$$\frac{\partial\gamma^O}{\partial\mu} = \frac{1}{8(\Lambda-1)}\frac{\sqrt{\frac{2-\Lambda}{1-\Lambda}(-8\mu-5\pi_g+8\mu\pi_g+3\mu\pi_n+5)^2}}{(-8\mu-5\pi_g+8\mu\pi_g+3\mu\pi_n+5)(\Lambda-2)^2}$$

$$\times\left(80\pi_g+27\pi_n-64\pi_g^2+16\pi_g^3+24\mu\pi_n-43\pi_g\pi_n-19\mu\pi_n^2+16\pi_g^2\pi_n+16\mu\pi_g\pi_n^2+16\mu\pi_g^2\pi_n-40\mu\pi_g\right.$$

The first term that needs to be signed is

$$\left(-8\mu - 5\pi_g + 8\mu\pi_g + 3\mu\pi_n + 5\right)$$

which is decreasing in μ. But

$$\left[\left(-8\mu - 5\pi_g + 8\mu\pi_g + 3\mu\pi_n + 5\right)\right]_{\mu=\frac{(1-\pi_g)(1+c)}{2-2\pi_g-\pi_n(1-c)}} = 2\left(1-\pi_g\right)\left(1-4c\right)\frac{1-\pi_g-\pi_n}{2-2\pi_g-\pi_n\left(1-c\right)} > 0$$

and so this is positive over our interval of interest. Now

$$\big(80\pi_g + 27\pi_n - 64\pi_g^2 + 16\pi_g^3 + 24\mu\pi_n - 43\pi_g\pi_n - 19\mu\pi_n^2 + 16\pi_g^2\pi_n + 16\mu\pi_g\pi_n^2 + 16\mu\pi_g^2\pi_n - 40\mu\pi_g\pi_n$$

$$= \pi_n\left(-40\pi_g - 19\pi_n + 16\pi_g^2 + 16\pi_g\pi_n + 24\right)\mu - \left(1-\pi_g\right)\left(-48\pi_g - 27\pi_n + 16\pi_g^2 + 16\pi_g\pi_n + 32\right)$$

$$= \pi_n\mathcal{B}\mu - \left(1-\pi_g\right)\mathcal{C}$$

Now,

$$\mathcal{B} = \left(-40\pi_g - 19\pi_n + 16\pi_g^2 + 16\pi_g\pi_n + 24\right)$$

is decreasing in π_g so setting $\pi_g = 1-\pi_n$ (as large as possible) makes it as small as possible and we get

$$\left[\mathcal{B}\right]_{\pi_g=1-\pi_n} = \left[\left(-40\pi_g - 19\pi_n + 16\pi_g^2 + 16\pi_g\pi_n + 24\right)\right]_{\pi_g=1-\pi_n} = 5\pi_n$$

So the whole expression is increasing in μ. That is,

$$\left[\pi_n \mathcal{B}\mu - (1-\pi_g)\,\mathcal{C}\right]_{\mu=\frac{(1-\pi_g)(1+c)}{2-2\pi_g-\pi_n(1-c)}} > \pi_n \mathcal{B}\mu - (1-\pi_g)\,\mathcal{C}$$

and we get

$$\left[\pi_n \mathcal{B}\mu - (1-\pi_g)\,\mathcal{C}\right]_{\mu=\frac{(1-\pi_g)(1+c)}{2-2\pi_g-\pi_n(1-c)}} = 2\,(1-\pi_g)\left(48\pi_g + 23\pi_n - 4c\pi_n - 16\pi_g^2 - 16\pi_g\pi_n - 32\right)\frac{1-\pi}{2-2\pi_g}$$

where every component is obviously positive except for

$$48\pi_g + 23\pi_n - 4c\pi_n - 16\pi_g^2 - 16\pi_g\pi_n - 32$$

which is increasing in π_g. So setting $\pi_g = 1-\pi_n$ (as large as possible) makes it as large as possible and we get

$$\left[48\pi_g + 23\pi_n - 4c\pi_n - 16\pi_g^2 - 16\pi_g\pi_n - 32\right]_{\pi_g=1-\pi_n} = -\pi_n\,(4c+9) < 0$$

This proves that

$$0 > \left[\pi_n \mathcal{B}\mu - (1-\pi_g)\,\mathcal{C}\right]_{\mu=\frac{(1-\pi_g)(1+c)}{2-2\pi_g-\pi_n(1-c)}} > \pi_n \mathcal{B}\mu - (1-\pi_g)\,\mathcal{C}$$

But, then going back to $\frac{\partial \gamma^O}{\partial \mu}$ all component are positive except for $\Lambda - 1$ at the denominator which is negative and the expression $\pi_n \mathcal{B}\mu - (1-\pi_g)\,\mathcal{C}$ which we just studied. Hence $\frac{\partial \gamma^O}{\partial \mu} > 0$.

∎