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Plaintive Plaintiffs: The First and Last Word in Debates

Elena D'Agostino

Daniel J Seidmann

Nottingham Interdisciplinary Centre for Economic and Political Research
School of Politics, The University of Nottingham, Law & Social Sciences Building,
University Park, Nottingham, NG7 2RD
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Elena D'Agostino
University of Messina
edagostino@unime.it

Daniel J Seidmann
The University of Nottingham
daniel.seidmann@nottingham.ac.uk

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Elena D'Agostino* and Daniel J. Seidmann†‡

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Abstract

Plaintiffs/prosecutors present their evidence before defendants in common law trials. We analyze a model of trials with the following properties. If litigants share a common pool of evidence then they never prefer to present first, but may prefer to present second. However, litigants may prefer to present first if they have different available evidence because presenting first replicates the follower's ex ante optimal commitment. If litigants share available evidence then a litigant cannot prefer the option to choose the order after observing the available evidence over always presenting second; and it may prefer to always present second over having the option to choose the order.

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*Department of Economics, University of Messina, Messina, Sicily 98122, Italy; edagostino@unime.it

†School of Economics, Nottingham University; daniel.seidmann@nottingham.ac.uk

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1. Introduction

According to conventional wisdom, advocates who compete to persuade a listener should either try to present the first or the last argument in a debate: the first speaker may anchor a listener's interpretation of subsequent arguments; whereas the final argument may be influential if listeners only remember the last word.

While cognitive limitations are doubtless important in some debates, the order of presentation can be significant for strategic reasons, as instanced by Obama's tactics in the second Presidential debate in 2012. Romney's claim that 47% of voters pay no income tax had been leaked before the debate.[1] Romney had presumably prepared a response; but Obama's success in the debate turned on his decision not to raise the issue till his concluding remarks.[2] On the other hand, going early may be advantageous. According to David Axelrod, Obama's 2012 campaign manager, the decision to start running ads early contributed to the campaign's success;[3] though research by Sides and Vavreck (2014) suggests, per contra, that back-loading was more effective. Finally, FOMC chairs have selected alternative orders: Greenspan chose to speak and to vote first at FOMC meetings (whereas Bernanke spoke last).[4] We explore the strategic consequences of the first and the last word in debates by analyzing a game-theoretic model in which the listener is not cognitively limited.

Debates take many forms; so, for the sake of concreteness, we focus on common law trials, whose procedural rules determine when litigants can speak and what they cannot say. Specifically, the plaintiff(s)/prosecution present their evidence, followed by the defendant(s); payoffs are well-defined; and the role of experienced attorneys and judges suggests that equilibrium plausibly predicts choices.[5] We ask whether/when the conventional order of presenting evidence serves the interests of either litigant or, for that matter, of the judge/jury?

We pose two variants on this question. First, could a litigant who did not yet know the available evidence gain from reversing the order? Second, what value would a litigant place on an option to choose the order after observing its available evidence? Most of our results address the first question, whose model we now sketch:

Nature starts the game by choosing the facts of the case (the *state*) and the evidence available to each litigant. Each litigant privately observes its available evidence before the trial starts, where the 'available evidence' is a collection of subsets of states (aka *witnesses*), each subset containing the realized state; and any collection of available witnesses is also available.

Litigants present their evidence in sequence. Each litigant has a single opportunity to present, when it can either pass or publicly present some witness(es) not hitherto presented:

[1] Mother Jones: "SECRET VIDEO: Romney tells millionaire donors what he REALLY thinks of Obama voters", Sept. 17, 2012.

[2] Slate: "When Candidates Attack: Obama won Tuesday night's slugfest, but Romney will live to fight another day", Oct. 17, 2012.

[3] In Axelrod's words: "We defined the race and Governor Romney before the conventions, and he was digging out of that hole for the remaining two months."

[4] See, for example, Blinder (2009).

[5] Exclusion rules are arguably motivated to minimize the effect of cognitive limitations.

a condition which we dub *no recall.* J observes the evidence presented by each litigant, and ends the game by either acquitting or convicting. The first litigant to present (the leader) bears a *burden of proof*: J rules against it if no evidence is presented.

There are two possible orders of presentation: the defendant (D) presents before the prosecution/plaintiff (P) or conversely; we treat these two orders as different games. Both litigants have state-independent preferences: D always wants J to acquit, P always wants J to convict; whereas J wants to avoid miscarriages of justice.

We analyze these games by characterizing the outcomes at pure strategy Bayes perfect equilibria where the burden of proof is met. There are typically several such equilibrium outcomes. Accordingly, we say that a player prefers to play one game (or prefers an order) if and only if it uniformly prefers the equilibrium outcomes of that game over the equilibrium outcomes of the other game at every pair of (available) evidence sets. The criterion is partial: a given player might not prefer either order. However, litigants' conflicting interests over verdicts translate into conflicting interests over the order: if D prefers to present first [resp. second] then P must prefer to present first [resp. second].

Consider an evidence set at which the same two witnesses, e and f, are available to each litigant. We represent play at this evidence set when D presents first in Table 1 below:

			P		
		e	f	ef	*pass*
	e	$\times$	α	$\times$	α
D	f		$\times$	$\times$	
	ef	$\times$	$\times$	$\times$	

Table 1

D chooses a row (in public), P responds by choosing a column, and J then reaches a verdict. We incorporate the burden of proof condition by precluding *pass* as a possible report for D, the first litigant to present. Evidence ef represents the combination of witnesses e and f, which is, by assumption, also always available to litigants. The $\times$s represent evidence pairs which are precluded by no recall: for example, P cannot present ef in response to D presenting e because ef requires witness e to be recalled. The remaining cells each contain the verdict that J would reach after observing the evidence presented by the two litigants, as illustrated by the two αs in the e-row, where α represents acquittal. These verdicts would be determined by the rules of the game if a combination of witnesses were only available in states where D is factually innocent (or factually guilty); we otherwise refer to evidence as *ambiguous***.** The verdict that J reaches after observing ambiguous evidence might be determined by equilibrium play at other evidence sets which contain the same evidence.

We first consider preferences over the order in a benchmark case where the same evidence is always available to both litigants**:** a case we call *discovery.* (This case is illustrated in Table 1 above.) We show that no litigant can prefer to present first in a discovery game. In particular, if D is acquitted at some evidence set when it leads then D is also acquitted at that evidence set in an equilibrium of the game in which it follows. We can use Table 1 to provide a partial intuition:

If D is acquitted at some evidence set after presenting first then J must acquit at every cell in that row which is not an $\times$, else P could profitably deviate and secure a conviction, as illustrated in Table 1 for the e-row. Now suppose that, in the other game (where P leads), J also acquits after D has presented e and after D has passed in response to all evidence which contains e (and is therefore represented by $\times$ in Table 2):

		P: e	f	ef
	e	$\times$	α	$\times$
D	f		$\times$	$\times$
	ef	$\times$	$\times$	$\times$
	$pass$	α		α

Table 2

Given this supposition, D's best response always secures acquittal: so J would also acquit when P presents first if it acquits when D leads.

We describe the intuition above as partial because it is not, in general, possible to retain the same verdict at every evidence pair after changing the order in which litigants present. Instead, we prove the result by constructing (equilibrium) strategy combinations for the game in which P leads: J observes the same evidence pair at every evidence set where it acquits when D leads; and P presents all of the available evidence at every other evidence set. D is therefore acquitted at every evidence set where it would be acquitted when leading; so, using our criterion for preferring an order, D cannot prefer to present first.

We also prove by example that litigants in discovery games could prefer to follow. Table 3 below illustrates how this works at evidence set E:

		P: e	f	ef	$g\_$	$pass$
	e	$\times$	γ	$\times$	α	
D	f		$\times$	$\times$	α	
	ef	$\times$	$\times$	$\times$	α	
	$g\_$	α	α	α	$\times$	

Table 3

Here γ represents conviction, while $g\_$ represents all evidence which contains a witness (g) who is only available when D is factually innocent, so every cell in a $g\_$ row or column contaisn an α. As E contains unambiguous evidence of factual innocence, D must be acquitted at this evidence set in every equilibrium of both games. Now suppose, in addition, that litigants can present e at exactly one other evidence set (say, F), where no other evidence is available; and that J would rationally acquit if the leader presented e and the follower passed both at E and at F, and would convict if it knew that F was the realized evidence set. D could not present e at E as leader because P could then secure conviction by responding with f, and J would acquit if D presented g; so, by construction, J must convict at F. However, P could present e at both E and F in an equilibrium when it leads. J acquits if D then passes; but P cannot profitably deviate at E because J would acquit whenever a litigant presents witness g. Hence, D may prefer to follow because of the acquittal that it then secures in an equilibrium at F. The underlying mechanism is that presenting second allows a litigant to keep its powder dry in an equilibrium.

Examples of this sort require evidence sets to intersect in particular ways. (E contains F in the example above.) We supplement our example by providing necessary conditions on the intersection of evidence sets for litigants to prefer to present second in discovery games.

We then present three reasons why litigants could prefer to present first when some evidence is exclusively available to a single litigant. Two of these reasons apply to cases in which each litigant always knows which evidence is available to its rival. Such cases arise naturally in common law trials because P cannot force D to testify, even if it knows, but cannot prove that the defendant is factually guilty; and J is not allowed to draw an adverse inference from D's decision not to testify.[6] Accordingly, we refer to such cases as *no-subpoena*.

First, as J does not observe which witnesses are available to each litigant, it must reach the same verdict after observing the same evidence pair. As litigants always share available evidence in discovery games, litigants could present the same evidence, irrespective of the leader's identity. This is not generally true in no-subpoena games. As the leader must meet a burden of proof, pooling is only possible if some evidence is available *to the leader* at two evidence sets: so the identity of the leader may then matter. We demonstrate by example that litigants may then prefer to present first. On the other hand, the burden of proof could create a reason why litigants prefer to be the follower which is absent in discovery games.

Second, the fact that some evidence is initially available to a given litigant may inform J about the state in no-subpoena games; and a litigant can only prove availability by presenting that evidence. Given our no recall assumption, a leader who presents a witness who is available to both litigants prevents the follower from proving availability, generating a preference for presenting first. Perhaps more surprisingly, a variant on this effect can generate a reason for preferring to present second which is absent in discovery games, where litigants are commonly known to share available evidence.

The third reason for preferring to present first applies to games in which a litigant is uncertain of its rival's available evidence. Prima facie, resolution of this uncertainty seems to reinforce the advantage of presenting second. This intuition turns out to be wrong: we show that a litigant who is uncertain of its rival's available evidence may prefer to lead.

The examples which we use to illustrate these three reasons have a common feature. The flexibility gained by responding on a case-by-case basis (viz. after observing its available evidence) may be ex ante disadvantageous because best responses at one evidence set may impose negative externalities at other evidence sets. Loosely speaking, the follower might ex ante gain by committing to its strategy. We formalize this intuition by analyzing

[6] *Griffin* v. *California* 380 U.S. 609 (1965).

a game in which the follower commits to its strategy before observing its available evidence. We show that the equilibrium outcomes of this commitment game coincide with the equilibrium outcomes of the game in which a litigant presents first in each of the three examples. In other words, litigants prefer to present first because play then replicates the effect of prior commitment.

Finally, we adopt an ex post perspective to the choice of order, focusing on situations in which litigants share the available evidence. Specifically, we analyze a game in which some litigant (say, D) first observes the available evidence and then chooses whether to present first. We call this the *ex post order game*. We prove that every equilibrium outcome of a game with a fixed order is an equilibrium outcome of the associated ex post order game, and that D cannot prefer to play the ex post order game to always presenting second. We then show by example that a litigant may prefer to always present second than to play the ex post order game.

We survey the related literature in Section 2, present our model of trials in Section 3, and analyze play in discovery games in Section 4. Section 5 considers games which are not played under discovery; Section 6 interprets the results therein by analyzing commitment games. Section 7 characterizes play in ex post order games, and assesses the advantages of an option to choose the order. We summarize our results in Section 8, and then consider their applicability to debates which lack some features of trials such as different stopping rules. We also show that a litigant may prefer to debate with or indeed delegate presentation to a rival than to alone present evidence.

2. Related literature

Our model is part of a literature on persuasion games, and is particularly related to papers which study debates in which litigants cannot directly prove the facts at issue:

The literature on persuasion games started with Milgrom (1981), who shows that a single litigant with state-independent preferences over the verdict separates if every state can be directly proved. Milgrom and Roberts (1986) extend this canonical model in two relevant directions. First, suppose that J is a priori uncertain of the evidence available to a single litigant. J could then not draw skeptical inferences which penalize presentation of imprecise evidence, and might therefore not learn the available evidence in any equilibrium. Milgrom and Roberts also study debates, in which litigants with conflicting preferences over the verdict simultaneously present evidence to a judge, who is modelled as an automaton. Competition between the litigants may then reveal the state, even if J cannot draw rational inferences. Our model combines these features by studying a debate in which a rational J is a priori uncertain of the evidence available to the litigants.[7]

Lipman and Seppi (1995) study sequential debates in which litigants observe the state and share available evidence, which need not contain an unambiguous witness.[8] Lipman and Seppi also allow both litigants to present messages from a rich cheap talk language in any fixed number of rounds, and for J to reach any number of verdicts. They construct

[7]We follow (most of) the subsequent literature by focusing on debates in which litigants present in sequence.

[8]We do not require that litigants observe the state.

an equilibrium which separates across states when litigants can present any combination of available witnesses: a condition which we also impose, and which they dub Full Reports (aka normality). In contrast to Lipman and Seppi, we exploit our focus on trials to impose a burden of proof on the leader, which excludes (cheap talk) speeches and therefore precludes Lipman and Seppi's construction. Nevertheless, we prove that all discovery games have an equilibrium which separates across evidence sets (rather than states), and demonstrate that non-discovery games need not have such separating equilibria. Aside from the modelling difference, this paper has a distinctive focus on the presentation order, for which Lipman and Seppi's model is inappropriate in the following sense. The equilibrium correspondence of a model in which speeches are allowed strictly contains the equilibrium correspondence of our (discovery) model because J can ignore all cheap talk messages. Players cannot prefer an order if the two games share two or more equilibrium outcomes; so allowing speeches would preclude any player preferring an order when litigants share available evidence. We discuss the effect of allowing several presentation rounds in Section 8.2, and of multiple verdicts in Section 8.4.

In Glazer and Rubinstein (2001), a state is a quintuple of aspects, each of which is coded for one litigant (and against the other litigant). Each witness reports the set of states which share the coding for one aspect; and the evidence available to a litigant in some state is the set of witnesses who report an aspect coded for that litigant. Thus, contrary to our model, Full Reports fails; and, as both litigants observe the state, a sequential debate is a no-subpoena game. Glazer and Rubinstein show that a J who could commit to the verdict it reaches after observing any evidence pair would prefer the litigants to present in sequence than simultaneously, would prefer that two litigants present two witnesses than that a single litigant presents two witnesses, and may commit to a strategy which violates Debate Consistency: the verdict depends not only on the evidence jointly presented but also on which litigant presented a given witness. The outcome of the optimal mechanism can also be realized in an equilibrium of a game in which J cannot commit: a property which also holds in discovery games because they have separating equilibria, which implement the outcome of J's optimal mechanism. These separating equilibria may also violate Debate Consistency, even though we impose Full Reports. In contrast to Glazer and Rubinstein, the mechanisms we consider vary the order of presentation, and our main results concern litigant preferences. Furthermore, our criterion leads us to consider the entire equilibrium correspondence.

Chen and Olszewski (2014) study a sequential debate with two equiprobable states and a continuum of verdicts. Litigants privately observe up to two signals, which are correlated with the state (rather than evidence) and commit ex ante to their strategies in sequence: where a strategy specifies which signal realization to (truthfully) present when two signal(s) are available: so Full Reports fails. Chen and Olszewski show that the equilibrium under commitment is also an equilibrium of the related game in which neither litigant can commit. Our main model precludes any ex ante commitment, but we demonstrate that non-discovery games in which the follower alone may commit to its strategy and in which the order is reversed may have the same equilibrium outcome correspondence.

Various other papers on debates are more tangentially related. In particular in Shin

(1994), litigants simultaneously present their privately observed available evidence, and J can reach a finite number of verdicts. Shin interprets the weight that the jury places on the evidence presented by each litigant in equilibrium as a burden of proof. By contrast, the burden of proof is an exogenously given stopping rule here. Sobel (1985) differs in further respects, including the absence of ambiguous evidence. However, the burden of proof in our sense functions as a refinement in Sobel (1985).

Nature chooses the evidence available to litigants in our model, contrary to Bayesian persuasion games like Gentzkow and Kamenica (forthcoming), where initially uninformed litigants choose the distribution of available evidence at each state, all of which is necessarily revealed. Nevertheless, our main criterion for preferring an order is also ex ante: we consider when a change in trial procedure would benefit players for every realization of available evidence. (We also consider the choice of order by a litigant who has observed its available evidence.)

Ottaviani and Sorensen (2001) consider the implications of changing the order in which litigants present cheap talk messages (rather than evidence).[9] In contrast to our model, the "state" also includes litigant competence, which they cannot observe; and litigant payoffs have a private (career concern) component. The ensuing herding effects play no role in our model.

The order in which litigants present at trial is neither constitutionally determined nor fixed by statute. Nevertheless, lawyers have rarely considered the effects of the order, and have then typically discussed it in terms of defendant rights in criminal trials.[10] Such rights cannot fully explain the order of presentation because defendants also present second in civil trials, where their rights are not privileged. Kozinski (2015) claims that presenting first might anchor beliefs; Damaska (1973) argues that any such advantage "pales to insignificance when assessed against the disadvantage of having to argue before knowing how the prosecution's case will develop." (fn 48, p.529)[11] This argument seems to conflate two apparently complementary arguments: presenting second allows a litigant to

- Tailor its presentation to the evidence presented by its rival when the available evidence is commonly known by litigants; and
- Respond to evidence which is unexpectedly available to its rival.

The first effect could apply in situations where litigants share the same available evidence; whereas the second effect requires asymmetric information. We demonstrate that these two effects may work in opposite directions. Indeed, we show that litigants who are uncertain of their rival's available evidence may therefore prefer to present first.

[9] In Hahn (2011), litigants present evidence.

[10] According to a standard argument, the prosecution presents first because defendants have a fundamental right to be presumed innocent, grounded in respect for persons. In particular, defendants are not required to justify themselves before an accusation has been made. (See, for example, Roberts and Zuckerman (2010) Ch 6.3.)

[11] Similarly, Williams (1963) suggests that the common law order may benefit defendants:

"The best reason for the English arrangement is that it enables the defending counsel to see how the prosecution case develops before deciding whether to put his client in the witness box." (p82)

The order in which witnesses testify before a grand jury has been more malleable: for example, the officer suspected of unlawfully killing Michael Brown in St. Louis testified first. Referring to that case, Cassell observed that "... it was to D's disadvantage to testify early. He'd be locked into a set of statements, and the grand jurors might later find inconsistencies."[12] This effect coheres with our results on discovery games.

While the effect of trial order has hitherto not been addressed game-theoretically, a literature in psychology (starting with Lund (1925)) asks how the order of presentation affects verdicts? The main difference is that we focus on the adversarial nature of common law trials. By contrast, mock jury trials (starting with Walker et al (1972)) fix the evidence available to each litigant and vary the order in which this evidence is presented - which nullifies the strategic effects which we address.[13] The underlying theory is rooted in individual psychology, referring to the workings of imperfect memory (whether primacy or recency effects predominate) and to priming or anchoring effects. Indeed, the theory seems to apply equally to the order in which the litigants present and the order in which a given litigant presents its evidence; our analysis, by contrast, only concerns the former case.

We conjecture that memory effects are less important when juries can deliberate (cf. Ellsworth (1989)) or when attorneys make closing statements than in mock jury trials. Indeed, if memory effects were important in real trials then one might expect significant differences between juries which could and couldn't discuss the evidence during the trial. Evidence from Arizona (which allowed discussion during civil trials) suggests no significant effects: cf. Hannaford et al (2000) and Diamond et al (2003).

3. Model (Fixed order)

We present our main model in this section. We describe the sequence in common law trials in Section 3.1, present our model in Section 3.2, and explain our criterion for preference over orders of presentation in Section 3.3.

3.1. Common law trials

Prior to a common law trial, litigants may be required to disclose available evidence to each other. These requirements are known as *discovery rules*.

Common law trials with a single defendant (D) and a single plaintiff/prosecutor (P) typically contain the following phases:

- P and then D make opening statements, which must be announcements of the evidence to be presented.
- P calls witnesses, who are cross-examined by D. (Speeches are not allowed.)

[12] Quoted in "Raised Hands and the Doubts of a Grand Jury" *New York Times* (11/29/14 page A1)

[13] Evidence from these experiments is mixed: for example, Kerstholt and Jackson (1998) report that earlier presentation can be beneficial or detrimental to the defendant, depending on the background information given to subjects. See also Costable and Klein (2005).

- D can present a motion to dismiss, on the grounds that P has not met its burden of proof.
- If the motion is dismissed then D calls witnesses, who are cross-examined by P. (Speeches are again not allowed.)
- P may be allowed to call new witnesses to rebut claims made by D's witnesses.
- P and then D make closing statements, which remind the judge/jury (J) of the evidence, and can suggest interpretations (aka argue the merits).
- J reaches a verdict.

In the next subsection, we present a model which captures most of these features.

3.2. The games

We analyze two games, which are distinguished by the order of presentation. Each game is played by D, P and J. Nature starts the game by selecting the facts of the case, and the evidence available to each litigant. The litigants then present evidence in sequence to J, which either acquits or convicts.

Game form

The game proceeds in four rounds:

Round 0 (Nature)

We describe the possible facts at issue (the *state*) as a countable set S with generic element s. S is partitioned into the states at which the defendant is factually guilty and the states at which it is factually innocent. Accordingly, we say that the *factual verdict* is guilt in the former set of states, and innocence otherwise. Nature chooses state s with a probability denoted by p_s; we refer to $\{p_s\}_{s \in S}$ as the *prior distribution*. Conditional on the realized state, Nature also chooses the evidence available to each litigant l, aka its *evidence set*, which we denote by E_l, and assume to be non-empty. We denote the (conditional) probability with which Nature chooses evidence sets E_D and E_P in state s by $\pi_s(E_D, E_P)$, referring to E_D, E_P as an *evidence set pair*. We write Σ for the union of the support of π_s across states s: that is, the set of possible evidence set pairs.

The evidence available to litigant l consists of pieces of evidence and combinations thereof. We think of each piece of evidence as indivisible, representing (for example) the testimony provided by a single witness; so we will henceforth refer to each piece of evidence as a *witness*. We identify each witness with a strict subset of states and, for expositional convenience, suppose that no more than one witness is associated with any given subset of states. If e and f are two witnesses then we write ef for evidence which combines the two witnesses, aka the composition of e and f.

We adopt two key assumptions about evidence sets:

First, witnesses are reliable and informative: their testimony always contains the realized state, and is a strict subset of S (viz. not cheap talk).[14] Testimony is therefore

[14] This implicitly supposes that cross-examination exposes deceit and/or that the prospect of being charged with perjury deters lying. Accordingly, litigants who testify are also assumed to report what they know.

evidence, in the sense introduced by Milgrom (1981).

Some evidence e may only be available at states with a single factual verdict. We then say that e directly proves that verdict. Similarly, we describe evidence which is only available at a single evidence set as directly proving that evidence set. It is convenient to refer to any evidence which directly proves a factual verdict and/or an evidence set as *unambiguous*, and to all other evidence as *ambiguous*. We will also say that such evidence induces a verdict.[15]

Second, if witnesses e and f are available to litigant l at some evidence set E_l then that litigant can also present ef; and, conversely, if a combination of witnesses is available, then so are all its nonempty subsets. We will refer to the combination of all witnesses available at E_l as the *full report* at E_l, and will call this assumption *Full Reports*. The full report at two distinct evidence sets must therefore differ.

Nature reveals E_l to litigant l, and possibly to its rival. (We will distinguish between complete and incomplete information games on this basis.) We assume, for expositional convenience, that no active player observes the state.[16]

Round 1 (Leader)

One of the litigants is designated as the *leader* (or litigant 1) and the other litigant as the *follower* (or litigant 2). Given its realized evidence set (say, E_1), the leader decides which witness(es) in E_1 to call.[17] We describe our assumption that the leader must present some evidence as the *burden of proof assumption*. A strategy for the leader is a selection from each of its possible evidence sets.

Round 2 (Follower)

Suppose that the leader has presented $e_1 \in E_1$ and that the follower's realized evidence set is E_2. The follower then decides whether to present some evidence or to pass (that is, to present no evidence): so the follower is not subject to a burden of proof. However, if the follower presents some evidence then we require that it only call witness(es) that were not presented by the leader, and must therefore pass if all of its available witnesses have already testified. We write this condition as $e_2 \in E_2 \backslash e_1$, and refer to it as the *no recall assumption***.** (Note that we abuse notation by including *pass* in $E_2 \backslash e_1$.) A strategy for the follower is a selection from $E_2 \backslash e$ for every realized evidence set pair and all evidence in E_1.

Round 3 (J)

Denote the evidence presented by the two litigants as e_1, e_2, and call it the *evidence pair*. After observing this pair, J ends the game by deciding whether to acquit (α) or convict (γ). A strategy for J is a selection $v \in \{\alpha, \gamma\}$ for every possible evidence pair. We will write $-v$ to denote the verdict other than v.

Payoffs

We suppose that the litigants only care about the verdict, and have conflicting preferences thereon: D [resp. P] earns 1 [resp. 0] whenever J acquits, and earns 0 [resp. 1] otherwise. Acquittal [resp. conviction] is therefore *favorable* to D [resp. P]. We write

[15] We will say that evidence pair e_1, e_2 induces a verdict if their composition ($e_1 e_2$) induces that verdict.

[16] This assumption will turn out to be immaterial for our analysis. The assumption has an interpretive advantage: prosecutors who know that the defendant is factually innocent are obliged not to prosecute.

[17] The leader's evidence set is non-empty; and each witness's testimony identifies a strict subset of states.

v_l for the verdict favorable to litigant l. J loses $1-d$ [resp. d] if it acquits at a factually guilty state [resp. convicts at a factually innocent state], and loses 0 otherwise, where $d \in (0,1)$ captures the standard of proof: that is, the posterior belief at which J is indifferent between acquittal and conviction.[18]

We write $v(\Phi)$ for the verdict ($v \in \{\alpha, \gamma\}$) which maximizes J's payoff if it knows that the evidence set pair is in some non-empty $\Phi \subseteq \Sigma$. We assume that $v(\Phi)$ is unique for every $\Phi \subseteq \Sigma$. We will write Σ_v for the evidence set pairs at which $v(E_D, E_P) = v$. We will write Φ_v for the collections of evidence sets Φ such that $v(\Phi) = v$.[19]

We define a *payoff vector* as a $\#\Sigma$-vector, each of whose elements specifies the payoff of each player at an evidence set pair in Σ.

Games

Our assumptions above define two games, depending on the identity of the leader: games which are distinguished by the order of presentation. If D presents first then we denote the game by $\Gamma^{D,P}$; if P presents first then we denote the game by $\Gamma^{P,D}$. (We will sometimes use notation $\Gamma^{1,2}$ or $\Gamma^{l,m}$, which should be interpreted on similar lines.)

Common law trials

The game is deliberately constructed to incorporate various of the features of common law trials described in the last subsection:

- Our (burden of proof) assumption captures one account of the burden of proof in common law trials: the follower (conventionally, D) can ask the judge to dismiss the case after Round 1 if the leader has passed.[20]

- Ou no recall assumption reinforces the requirement that evidence be relevant: the follower has already had the opportunity to cross-examine the leader's witnesses.[21]

- Litigants are allowed to present as many relevant witnesses as they like in common law trials. In this sense, Full Reports seems to be a natural assumption here.

- Our assumption that litigants must either present evidence or pass captures the requirement that litigants can only make speeches (closing statements) when the evidence has been presented. We exclude closing statements because our assumptions about litigant preferences imply that these statements could not change J's beliefs.

- In common law trials, the leader can only present rebuttal evidence if one of the follower's witnesses has given surprise testimony. Our assumption that the game ends after Round 2 may again be justified on the understanding that interrogation and cross-examination reveal all that a witness knows.[22]

[18] The standard of proof is reasonable doubt in criminal trials, and the balance of probabilities in civil trials.

[19] Σ_v is the collection of singleton pairs such that $v(E_D, E_P) = v$.

[20] On an alternative account, P has the burden, irrespective of when it presents. This rule would allow a first-presenting D to ensure acquittal, under discovery, by presenting all of the available evidence. See Spier (2007) for further discussion.

[21] Given no recall, we must allow the follower to pass.

[22] We will return to the implications of other stopping rules in Section 8.2.

Solution concept

Our solution concept requires us to specify beliefs (viz. a probability distribution over Σ) for each player about the history of play that led to an information set controlled by that player. We solve each game by characterizing its *equilibria*: that is, the pure strategy combinations and beliefs which satisfy the following conditions, where a player's beliefs are defined at the information sets it controls:

1. Given its beliefs and other players' strategies, no player can profitably deviate after any history;

2. Each player's belief satisfies Bayes rule whenever possible, given the strategy combination;

3. J reaches verdict v whenever a litigant presents evidence which induces v.

We require condition 3 to hold off, as well as on the equilibrium path. Lipman and Seppi (1995) call this condition 'feasibility'.

We will refer to the payoff vector reached on an equilibrium path as an *outcome*. We write $w^{D,P}$ [resp. $w^{P,D}$] for the set of outcomes in $\Gamma^{D,P}$ [resp. $\Gamma^{P,D}$]. We will denote strategy combinations by capital letters (e.g. X) and the associated outcome by small letters (e.g. x).

We say that an equilibrium is *separating* if it prescribes different evidence pairs at different evidence set pairs (rather than in different states). By definition, J must then reach verdict $v(E_D, E_P)$ at every evidence set pair E_D, E_P. We also say that a (non-separating) equilibrium prescribes a *wrongful acquittal* at E_D, E_P if it prescribes acquittal at that evidence set pair and $v(E_D, E_P) = \gamma$; we define *wrongful convictions* analogously. An equilibrium prescribes a *miscarriage of justice* at E_D, E_P if it prescribes either a wrongful acquittal or a wrongful conviction at that evidence set pair.

3.3. Preferences over orders of presentation

Our main results will explore the conditions under which a player prefers to present first or to present second. We interpret this as a question about the player's preferences over the set of outcomes of the two games because each game typically has multiple outcomes, as illustrated by the following example:

Example 1 *There are three states: $S = \{m, o, mo\}$, where mo is the only factually guilty state. There are two witnesses:*

$$e = \{m, mo\} \text{ and } f = \{o, mo\}.$$

and three evidence sets:

$$M = \{e\}, O = \{f\} \text{ and } MO = \{e, f\}.$$

In every state, the two litigants share the same evidence set (so we write $E_D = E_P \equiv E$). The conditional distribution of this evidence set is

$$\pi_m(M) = \pi_o(O) = \pi_{mo}(MO) = 1.$$

The prior distribution satisfies

$$\frac{p_{mo}}{p_{mo}+p_m} < d < \frac{p_{mo}}{p_{mo}+p_o}. \blacksquare$$

Example 1 can be interpreted as follows. D is commonly known to have motive and/or opportunity to commit the crime, but can only be convicted if J believes that it has both. There are two factually innocent states: in one ($s=m$), D only has motive, in the other factually innocent state ($s=o$), D only has opportunity. Both witnesses are ambiguous: each could be available in a factually innocent and in a factually guilty state. In state m, litigants can prove and only prove motive by presenting e; in state o, litigants can prove and only prove opportunity by presenting f. Finally, in the factually guilty state, each litigant could present both witnesses (ef: the full report at MO), thereby inducing conviction, or could prove either motive or opportunity.

Proposition 1 below will imply that both games possess a separating equilibrium, in which J acquits unless the evidence set is MO. Both games have another equilibrium which prescribes: the leader to present e at M and to present f otherwise; P to present f [resp. e] at MO in response to D presenting e [resp. f] in $\Gamma^{D,P}$; D to always pass in $\Gamma^{P,D}$; and J to acquit after and only after observing $e, pass$. The two outcomes only differ at O, where the non-separating equilibrium alone prescribes a wrongful conviction. If we selected the separating equilibrium in $\Gamma^{D,P}$ and the other equilibrium in $\Gamma^{P,D}$ then each litigant would prefer to present first; and each litigant would prefer to present second if we reversed the selection.

This selection problem applies much more generally. Moreover, standard refinements do not reduce the multiplicity of outcomes because each litigant has the same preference ordering over J's actions at every evidence set pair. Accordingly, we now provide a criterion for preference over multiple outcomes which does not rely on selection arguments.

We start by defining preferences of player $q \in \{D, P, J\}$ over payoff vectors x and y. We write $x_q(E_D, E_P)$ and $y_q(E_D, E_P)$ as the payoff that X and Y respectively prescribe player q to earn at evidence set pair E_D, E_P.

We consider two alternative criteria. First, we say that player q *ex post* strictly prefers x over y at an evidence set pair E_D, E_P **if** $x_q(E_D, E_P) > y_q(E_D, E_P)$. A given player could ex post prefer x over y and y over x at different evidence set pairs. This is impossible on our second criterion:

We say that player q *ex ante* strictly prefers x over y (denoted by $x \succ_q y$) when the following two conditions hold:

- There is an evidence set pair E_D, E_P at which $x_q(E_D, E_P) > y_q(E_D, E_P)$;
- There is no evidence set pair E_D, E_P at which $x_q(E_D, E_P) < y_q(E_D, E_P)$.

Player q *ex ante* weakly prefers vector x over the vector y (denoted by $x \succeq_q y$) if the latter condition alone holds.

If $x \neq y$ then player q ex ante strictly prefers x over y if and only if it ex ante weakly prefers x over y. Ex ante weak and strict preference each define a partial ordering over payoff vectors.

Each of these criteria define a transitive ordering over payoff vectors: a feature which we will exploit below.

We now use these criteria for preferences over outcomes to define preferences over sets of outcomes. We say that *player q ex post prefers litigant $l \neq m$ to present first* at an evidence set pair E_D, E_P if $w^{P,D}$ and $w^{D,P}$ are non-empty and

Condition 1 (ex post) $x_q(E_D, E_P) \geq y_q(E_D, E_P)$ for every $x \in w^{l,m}$ and every $y \in w^{m,l}$; and

Condition 2 (ex post) $x_q(E_D, E_P) > y_q(E_D, E_P)$ for some $x \in w^{l,m}$ and some $y \in w^{m,l}$.

We also say that player q *ex ante prefers litigant $l \neq m$ to present first* if

Condition 1 (ex ante) $x \succeq_q y$: for every $x \in w^{l,m}$ and every $y \in w^{m,l}$; and

Condition 2 (ex ante) $x \succ_q y$: for some $x \in w^{l,m}$ and some $y \in w^{m,l}$.

Our criteria define a partial ordering over games: in particular, no player either ex ante or ex post prefers an order (at any evidence set) in Example 1 above. More generally, no player can either ex ante or ex post prefer an order (at any evidence set pair) if both games share two or more distinct outcomes. In other words, our criteria are hard to satisfy: which cuts in favor of our negative results, and against our positive results.

As litigants have opposing preferences over the verdict, P prefers to present first [resp. second] if and only if D prefers to present first [resp. second] on each criterion.

If either pair of Conditions holds then player q's preference over the two games does not depend on which equilibrium is selected in each game.

If player q ex ante prefers an order then it ex post prefers that order at some evidence set pair, but the converse is false. We will return to this feature in the next section.

4. Discovery games

Discovery rules require litigants to share their available evidence, and are enforced by means such as depositions, interrogatories and motions to inspect and copy documents. Discovery rules were introduced in federal civil trials in 1938 to improve fact-finding and to prevent litigants from introducing surprise evidence at trial (cf. Subrin (1998)).[23] In this section, we study preferences over the order of presentation when discovery rules apply to both litigants. Our model allows us to address this case by imposing conditions on Σ: the support of evidence set pairs. Specifically, we will describe games in which $E_D = E_P$ for every $E_D, E_P \in \Sigma$ as *discovery games*.[24] We will focus on discovery games throughout this section.

We can simplify the notation in discovery games. If $E_D = E_P = E$ then we denote the evidence set pair as E, and write $v(E)$ for the verdict that J would reach if it knew that E had been realized. We define the collection of all available witnesses at an evidence set E as the *full report* at E. It will be useful to say that the follower *completes* the full report at E if $e_1 e_2$ equals the full report or if the leader presents the full report (so the follower must pass).

[23] Bone (2012) provides further details, and reviews the law and economics literature.

[24] There are two possible interpretations: either litigants only observe their own available evidence but know that evidence sets are perfectly correlated across litigants; or litigants directly observe both evidence sets (which are, in fact, always identical).

We will describe an evidence set which is contained in no other evidence as *maximal.* The full report at any maximal evidence set E induces $v(E)$. We define Σ_v as the evidence sets E such that $v(E) = v$.

We start by providing a result of independent interest, which we will exploit below:

Proposition 1 *Every discovery game has a separating equilibrium.*

Proof Consider the following construction:

- At each evidence set E: the leader presents the full report;
- At each evidence set E: the follower responds to $e_1 \in E$ by completing the full report unless $v(E) = v_1$ and $e_1 \in F \subset E$ such that $v(F) = v_2$, in which case the follower presents $F \backslash e_1$. (Recall that v_l is the verdict favorable to l);
- After observing e_1, f (where $f = e_2$ or *pass*), J believes that the realized evidence set is: E if $e_1 f$ is the full report at E; is in Σ_v if e_1, f induces verdict v; and is otherwise in Σ_{v_1};
- After observing e_1, f (where $f = e_2$ or *pass*), J reaches verdict: $v(E)$ if $e_1 f$ is the full report at E; v if e_1, f induces v; and v_1 otherwise.

This strategy combination implies that J best responds both to the evidence presented on the path and after the leader has deviated, as well as to any evidence pair which induces a verdict. Any other evidence pair is ambiguous; so J cannot profitably deviate.

If $v(E) = v_1$ then the leader cannot profitably deviate at any evidence set E because presenting the full report results in its favored verdict; and if $v(E) = v_2$ then the leader cannot profitably deviate because it anticipates that J will reach v_2 after the follower's response. The follower cannot profitably deviate because its response to the leader presenting some, but not all available witnesses at E results in J reaching its favored verdict whenever $v(F) = v_2$ for some $F \subset E$ which contains e_1.■

Proposition 1 implies that every discovery game has an equilibrium, even though we focus on pure strategy combinations. In the constructed equilibrium, J learns the evidence available (rather than the state), irrespective of the order, in every discovery game. According to the construction, the leader presents the full report at every evidence set; and J's strategy may satisfy Glazer and Rubinstein's (2001) notion of Debate Consistency: its verdict after e_1, e_2 only depends on $e_1 e_2$. However, discovery games may have equilibria which fail Debate Consistency.[25]

As $w^{P,D}$ and $w^{D,P}$ are non-empty in discovery games, we can use the criteria introduced in Section 3.3 to consider preferences over the order of presentation. Our main result in this section is

[25]For example, suppose that $S = \{1, 2, c, C\}$, where D is factually innocent in the numbered states; that the witnesses are $e = \{1, c\}$, $f = \{1, 2, c\}$, $g = \{c\}$, $h = \{C\}$ and $k = \{2\}$; that evidence sets are $E = \{e, f\}$, $F = \{k\}$, $G = \{e, f, g\}$ and $H = \{h\}$; and that $\pi_1(E) = \pi_2(F) = \pi_c(G) = \pi_C(H) = 1$. $\Gamma^{D,P}$ has a separating equilibrium in which J acquits after observing $e, pass$ at E: J would then acquit after observing e, f and convict after observing f, e.

Theorem 1 *In discovery games:*

a) *Litigants cannot ex post prefer to present first at any evidence set;*

b) *Litigants can ex ante prefer to present second; and*

c) *J only ex ante prefers an order if litigants ex ante prefer to present second.*

Theorem 1a) implies that litigants cannot ex ante prefer to present first, while Theorem 1b) implies that litigants may ex post prefer to present second.

Proof
Our proof relies on the following result:

Lemma *Let $\Gamma^{D,P}$ and $\Gamma^{P,D}$ be discovery games. Suppose that $\Gamma^{1,2}$ has an equilibrium which prescribes verdict v_1 exactly at the evidence sets $\Sigma_1 : 1 \in \{D, P\}$. $\Gamma^{2,1}$ then has an equilibrium which prescribes verdict v_1 at every evidence set in Σ_1.*

Proof If the equilibrium is separating then the result follows from Proposition 1. Accordingly, suppose otherwise. We focus on the case in which D presents first for expositional convenience. There is then a non-separating equilibrium of $\Gamma^{D,P}$ (say, X) which partitions Σ into Σ_α^X and Σ_γ^X, where X prescribes D and P to respectively present $e_1^X(E)$ and $e_2^X(E)$ and for J to reach verdict v after observing $e_1^X(E), e_2^X(E)$ at every $E \in \Sigma_v^X$. The full report at $E \in \Sigma_\alpha^X$ [resp. $F \in \Sigma_v^X$] cannot induce conviction [resp. acquittal], else P [resp. D] could profitably deviate to completing the full report.
Consider the following strategy combination (say, Y) and beliefs in $\Gamma^{P,D}$:

- Y prescribes P to present $e_1^X(E)$ at every $E \in \Sigma_\alpha^X$, and to present the full report at any other evidence set;
- At any evidence set in Σ_α^X: Y prescribes D to respond to $e_1^X(E)$ by presenting $e_2^X(E)$, and otherwise to respond to e_P by completing the full report at E;
- At any evidence set $F \in \Sigma_\gamma^X$: Y prescribes D to complete the full report at F unless there is e_D which does not complete the full report at F such that either
  - $e_P, e_D = e_1^X(E), e_2^X(E)$ for some $E \in \Sigma_\alpha^X$ or
  - e_D completes the full report at some evidence set in Σ_α^X or some evidence set E at which $v(E) = \alpha$.

  In both of these cases, Y prescribes D to present e_D in response to e_P;
- After observing e_P, f (where f is either evidence or pass), J believes that the realized evidence set is in Σ_α if
  - $e_P, f = e_1^X(E), e_2^X(E)$ for some $E \in \Sigma_\alpha^X$ or

– $e_P f$ induces α or

– f completes the full report at some evidence set $E \in \Sigma_\alpha^X \cup \Sigma_\alpha$,

and that the realized evidence set is in Σ_γ otherwise.

Y prescribes J to acquit if and only if it believes that the realized evidence set is in Σ_α.

Y cannot prescribe the same evidence pair on the path at any $E \in \Sigma_\alpha^X$ and any $F \in \Sigma_\gamma^X$. To see this, suppose per contra that X prescribes D to present the full report at F (say, F^*). X must then prescribe J to acquit after observing $F^*, pass$, else P could profitably deviate to passing at E. However, this implies that D could profitably deviate from X's prescription at F to presenting the full report because, by construction, X prescribes J to convict on the path at F.

J cannot profitably deviate, given the strategies which Y prescribes for litigants. To see this, note that Y only prescribes J to acquit after observing evidence which either induces α; or is played on the path at evidence sets $F \in \Sigma_\gamma^X$ which satisfy $v(F) = \alpha$; or is played on the path at evidence sets in Σ_α^X (so J holds the same posterior beliefs after observing $e_1^X(E), e_2^X(E)$ in both games); or if the evidence is ambiguous, such as the full report at $E \in \Sigma_\alpha^X \cap \Sigma_\gamma$. This evidence must be ambiguous, else P could profitably deviate from X by presenting the full report at E. Finally, Y never prescribes J to convict after observing evidence which induces acquittal.

We now argue that litigants cannot profitably deviate at any evidence set:

Consider an evidence set $E \in \Sigma_\alpha^X$. Y prescribes P to present $e_1^X(E)$ and J to acquit after observing either $e_1^X(E), e_2^X(E)$ or D completing the full report at E. D can therefore secure acquittal, irrespective of the evidence that P presents at E; so no litigant can profitably deviate from Y's prescription at E.

Now consider an evidence set $F \in \Sigma_\gamma^X$. If $v(F) = \alpha$ then Y prescribes J to acquit whenever D completes the full report at F; so neither litigant can profitably deviate. If $v(F) = \gamma$ then P cannot profitably deviate from presenting the full report; and if P deviates to presenting e_P then D can profitably deviate from completing the full report if and only if there is e_D such that J acquits after observing e_P, e_D: to wit if the evidence pair is prescribed by Y or completes the full report at some $E \in \Sigma_\alpha^X$.

These arguments imply that Y is an equilibrium in $\Gamma^{P,D}$. Y prescribes acquittal at every evidence set in Σ_α^X, confirming the result for cases in which X describes an equilibrium for $\Gamma^{D,P}$. An equivalent argument establishes Lemma for the other case.■

a) follows from Lemma. To see this suppose, per contra, that D ex post prefers to present first at E. Condition 2 (ex post) then implies that $\Gamma^{D,P}$ has an equilibrium X which prescribes acquittal at E, while $\Gamma^{P,D}$ has an equilibrium Y which prescribes conviction at E. Lemma then implies that $\Gamma^{P,D}$ has another equilibrium Y' which also prescribes acquittal at E, while $\Gamma^{D,P}$ has another equilibrium X' which prescribes conviction at E.

In sum, we have $x, x' \in w^{D,P}$ and $y, y' \in w^{P,D}$ such that

$$y'_D(E) \geq x_D(E) > y_D(E) \geq x'_D(E).$$

These inequalities imply that $y'_D(E) > x'_D(E)$, contrary to Condition 1 (ex post). An analogous argument precludes P ex post preferring to present first.

b) We prove this part with an example in which one game only has a separating outcome, while the other game also has another outcome:

Example 2 *There are four states: $S = \{1, 2, 3, i\}$, and the defendant is only factually innocent in state i. There are three witnesses:*

$$e = \{1, 2, i\},\ f = \{2, 3, i\} \text{ and } g = \{i\}$$

and four evidence sets:

$$E = \{e, f, g\},\ F = \{e, f\},\ G = \{f\} \text{ and } H = \{e\},$$

whose conditional distribution satisfies

$$\pi_i(E) = \pi_2(F) = \pi_3(G) = \pi_1(H) = 1.$$

The prior distribution satisfies

$$\frac{p_1}{p_1 + p_i} < d < \min\{\frac{p_2}{p_2 + p_i}, \frac{p_3}{p_3 + p_i}\}. \blacksquare$$

These conditions imply that, at any non-separating outcome, litigants must present $e, pass$ at E and at H, and J must wrongfully acquit at H. To see this, note that g directly proves factual innocence, so J must acquit at E in any equilibrium; but the conditions on priors imply that J would convict were litigants to present the same evidence pair at E as at F and/or G.

Claim 1 *In Example 2, $\Gamma^{P,D}$ has an equilibrium with a wrongful acquittal.*

Proof Consider the following strategy combination and beliefs:

- P presents e at E, and presents the full report at every other evidence set;
- D passes in response to P presenting e (at E and at F), and otherwise completes the full report;
- J believes that the realized evidence set is
  - E and acquits if either litigant presents evidence which contains g [resp. f];
  - F or G and convicts if either litigant presents evidence which contains f but not g;
  - E or H and acquits after observing $e, pass$.

P cannot profitably deviate at E because D could secure acquittal by presenting g. The strategy combination and beliefs therefore form an equilibrium.■

Claim 2 *In Example 2, every outcome of $\Gamma^{D,P}$ is separating.*

Proof Any other outcome is supported by an equilibrium which prescribes D to present e and P to then pass at E and at H; and J to acquit after observing $e, pass$ and to convict at F. J must also acquit after observing e, f else P could profitably deviate to presenting f at E. This implies that D could profitably deviate to presenting e at F.■

Given the condition on priors in Example 2: Proposition 1 and Claim 1 imply that $w^{P,D}$ consists of a separating outcome and an outcome with a wrongful acquittal at H; while Proposition 1 and Claim 2 imply that $w^{D,P}$ only consists of a separating outcome. Consequently, both litigants must ex ante prefer to present second.

c) Suppose that J indeed ex ante prefers litigant $l \neq m$ to present first. J ex ante strictly prefers a separating outcome over any other outcome; and both games possess a separating outcome by Proposition 1. Consequently, J ex ante prefers $\Gamma^{l,m}$ over $\Gamma^{m,l}$ if and only if $\Gamma^{m,l}$ alone has a non-separating outcome.

No non-separating equilibrium of $\Gamma^{m,l}$ can prescribe J to wrongfully reach verdict v_m at any evidence set E because Lemma would then imply that $\Gamma^{l,m}$ also has a non-separating equilibrium, contrary to our supposition that J prefers an order. As every non-separating equilibrium of $\Gamma^{m,l}$ only prescribes J to wrongfully reach verdict v_l, litigants prefer to present second.■

The criterion which we introduced in Section 3.3 compares equilibrium correspondences, and therefore seems rather unwieldy. Theorem 1 provides a strikingly clean description of preferences over the order without fully characterizing these correspondences.

Presenting second allows the follower to condition the evidence it presents on the leader's choice at each evidence set. Example 2 illustrates why such flexibility may be advantageous in discovery games. The full report at E induces acquittal; so J must acquit at E in both games. However, the evidence which the leader presents at E determines J's verdict at H. In the non-separating equilibrium of $\Gamma^{P,D}$, P presents e at E because it anticipates that D would otherwise complete the full report. D cannot present e at E in $\Gamma^{D,P}$ as P would optimally respond by presenting f rather than passing because equilibrium play at F implies that J would convict after observing e, f. Flexibility therefore allows for pooling, resulting in a verdict favorable to the follower.

Now amend Example 2 by identifying states 1, 2 and 3 as factually innocent, and state i as factually guilty. Arguments used above then imply that J convicts after observing $e, pass$ at E and at H in an equilibrium of $\Gamma^{D,P}$, whereas $\Gamma^{P,D}$ only has separating equilibria. More generally, litigants prefer to present second when $w^{D,P} \cup w^{P,D}$ contains a non-separating outcome, every miscarriage of justice in an equilibrium of $\Gamma^{D,P}$ is a wrongful conviction and every miscarriage of justice in $\Gamma^{P,D}$ is a wrongful acquittal.

Notice, for future reference, that the proof of Theorem 1 does not rely on no recall.

We will now argue that the following condition, which generalizes Example 2, is necessary for litigants to prefer to present second.

Interlocking Evidence Sets (IES)

a Σ *can be partitioned into a collection of evidence sets* $\{\Phi^i\}$*, such that all evidence sets in* Φ^i *have a non-empty intersection. Some* $\Phi^i \in \Phi_v$ *intersect with* Σ_α *and with* Σ_γ*; and no evidence available at an evidence set in* Σ_{-v} *induces verdict* $-v$.

b *An evidence set in some* $\Phi^i \in \Phi_v$ *contains at least two witnesses, and intersects with an evidence set outside* Φ_v.

Proposition 2 below provides *some* necessary conditions for litigants to prefer to present second. If $\Gamma^{l,m}$ has an outcome which m strictly prefers over separation then we construct a strategy combination in $\Gamma^{m,l}$ which replicates the payoff vector in $\Gamma^{l,m}$, and provide conditions for this strategy combination not to be an equilibrium.

Proposition 2 *Litigants only prefer to present second in discovery games if IES holds.*

Proof Any equilibrium partitions Σ into collections of evidence sets, say Φ^i, such that the evidence sets in each Φ^i have a non-empty intersection; prescribes litigant l to present $e_l(E)$ at every evidence set $E \in \Phi^i$; and prescribes J to reach verdict $v(\Phi^i)$ after observing the evidence pair presented at each Φ^i. We write Φ^i_v for the collections at which J reaches verdict v according to a given equilibrium.

Proposition 1 implies that litigants can only prefer to present second if there is $l \neq m \in \{D, P\}$ such that $\Gamma^{l,m}$ has an equilibrium which prescribes v_m at some evidence set(s) in Σ_{v_l}, and does not prescribe v_l at any evidence set in Σ_{v_m}, while no equilibrium of $\Gamma^{m,l}$ prescribes v_m at some evidence set(s) in Σ_{v_l}.

We focus on the case where $l = P$: arguments for the other case are symmetric. Write X for the equilibrium of $\Gamma^{P,D}$ and Σ^X_v for the evidence sets at which X prescribes verdict v. Existence of X requires that there be collections, Φ^i_α, which contain evidence sets in Σ_γ. X must also satisfy $\Sigma^X_\gamma \subset \Sigma_\gamma$. Furthermore, no evidence which induces v_l can be available at any evidence set at which X prescribes v_m, else m could profitably deviate. This proves part a. We will show that, if X exists, then $\Gamma^{D,P}$ has an equilibrium with the same outcome unless the conditions in part b are also satisfied.

Take any such equilibrium X of $\Gamma^{P,D}$ and define a strategy combination Y for $\Gamma^{D,P}$ which prescribes the same verdicts at each evidence set as X. Specifically, Y prescribes

> D to present $e^X_P(E)$ at every evidence set E, and P to respond to each $e \in E$ by presenting the same evidence as X prescribes D to present in response to e;
>
> J to reach the same verdict after observing D and P presenting e and f respectively as X prescribed after observing D and P presenting f and e respectively.

As litigants prefer to present second, some player must have a profitable deviation from Y at *some* evidence set. This player can clearly not be J.

We start by considering putative profitable deviations from Y at some $E \in \Phi^i_\alpha$. Write $e^X_D(E)$ for the response that X prescribes to $e^X_P(E)$ at E. As Y prescribes acquittal, only

P could profitably deviate. P can only profitably deviate from Y at E if X prescribes J to convict after observing P and D respectively presenting $e_P^X(E)$ and some $f \in E$ other than $e_D^X(E)$ - where f could be *pass*. If X prescribes J to acquit at every evidence set which contains $e_P^X(E)$ and f then $\Gamma^{P,D}$ has another equilibrium, say X', which prescribes the same outcome as X. P can then not profitably deviate from the strategy combination Y' which is defined from X'. Consequently, P can only profitably deviate at some $E \in \Phi_\alpha^i$ which intersects with an evidence set in $\Sigma_\gamma^X \subset \Sigma_\gamma$.

Now consider putative profitable deviations from X at some evidence set $E \in \Phi_\gamma^i$. By construction, Y prescribes conviction at E, so P cannot profitably deviate. D can only profitably deviate if there is $e \neq e_P^X(E)$ such that Y prescribes acquittal after observing D present e and P present f: for every $f \in E$ which does not contain e. In particular, E cannot be a singleton evidence set. If e does not intersect with $e_P^X(E)$ then the deviation could not be profitable: for D could then profitably deviate from X to presenting e in response to $e_P^X(E)$. Accordingly, suppose that D deviates to some e whose intersection with $e_P^X(E)$ is non-empty.

If any ef is only contained in E then it must induce conviction; so feasibility implies that X (and therefore Y) must prescribe conviction whenever P responds to e by presenting f. Consequently, D can only profitably deviate to presenting e if ef is contained in another evidence set which is not in Φ_γ^i: for every $f \in E$ which does not intersect e. Thus, the requisite profitable deviation requires that some Φ_γ^i contains a non-singleton evidence set, two of whose elements are contained in an evidence set outside Φ_γ^i.

In sum, IES specifies necessary conditions for $\Gamma^{P,D}$ alone to have an outcome which D prefers over separation. Necessary conditions for $\Gamma^{D,P}$ alone to have an outcome which is worse for D than separation are symmetric; so IES provides necessary conditions for this case as well.■

No recall allows for the possibility that D can profitably deviate at an evidence set where J prescribes conviction. However, as noted above, our analysis of Example 2 does not rely on no recall. Dropping recall would still allow for litigants to prefer to present second.

IES does not preclude profitable deviations from Y at a collection Φ^i which consists of a single evidence set (say, E). However, this is impossible if X, the equilibrium of $\Gamma^{l,m}$, prescribes v_m at E: X must then prescribe J to reach v_m after l has presented the full report at E, else l could profitably deviate; so $\Gamma^{l,m}$ has an outsome-equivalent equilibrium X' which prescribes l to present the full report at E, and l can then not profitably deviate at E from the strategy combination in $\Gamma^{m,l}$ constructed from X'. This property aligns IES with Example 2, where J acquits in $\Gamma^{P,D}$ after observing the same evidence pair at E and at H.

Say that all evidence sets *are provable* if every $E \in \Sigma$ is maximal or contains (unambiguous) evidence which directly proves factual innocence or directly proves factual guilt. Proposition 2 then implies that players can only prefer an order in discovery games if some evidence set is not provable: that is, only contains ambiguous evidence. Example 4 in Section 5.1.2 will demonstrate that this property does not generalize to all no-subpoena games.

J is best off in a separating equilibrium, which exists in both games (by Proposition 1). Consequently, if J ex ante prefers litigant $l \neq m$ to be the leader then $w^{l,m}$ alone consists of the separating outcome. Example 2 instances a game in which J ex ante prefers an order (D as leader). The converse of Theorem 1c) is false: for litigants would prefer to present second if non-separating equilibria of $\Gamma^{l,m}$ only prescribe J to wrongfully reach verdict $v_m : l = D, P$; whereas J only ex ante prefers an order if exactly one of the games has a non-separating outcome.

We end this section with a couple of technical observations:

The proofs of Proposition 1 and of Theorem 1a) and 1b) all rely on constructed equilibria. In all of these constructions, J's strategy satisfies a Markovian property which implies Dynamic Consistency: the verdict which J reaches after observing any evidence pair e_1, e_2 only depends on $e_1 e_2$ and not, in particular, on which litigant presents e_1. Li and Norman (2015) apply such a refinement in a Bayesian persuasion game where litigants have access to the same signals. Their arguments for the plausibility of Markovian equilibria also seem pertinent in discovery games. We do not apply this refinement when defining preference over orders because it would weaken Theorem 1a). However, the outcomes in the proofs of Claims 1 and 2 can be supported by equilibria which satisfy the Markovian property; so Theorem 1b) could be strengthened.

Lemma is phrased in terms of discovery games. However, the following property is sufficient for litigants not to ex ante prefer to present first in any game: $x \in w^{1,2}$ implies that there is $y \in w^{2,1}$ such that $x \preccurlyeq_1 y$. In the next section, we will illustrate non-discovery games where litigants ex ante prefer to present first, and therefore where this generalization of Lemma fails.

5. Non-discovery games

In the last section, we demonstrated that litigants cannot prefer to present first, but may prefer to present second in discovery games if IES is satisfied. In this section, we consider games which are not played under discovery. We provide examples in which litigants prefer to present first, as well as examples of non-discovery games in which litigants prefer to present second, even though evidence sets do not interlock. We divide this section into two parts, each of which represents a failure of the discovery assumption.

5.1. No-subpoena games

Discovery rules require litigant l not only to make its rival (m) aware of l's available evidence, but also to allow m to call the witnesses available to l. In this subsection, we consider games in which the evidence available to each litigant is commonly known by both litigants (but not J); but, in contrast to discovery games, m cannot call witnesses who are only available to litigant l. We refer to such cases as *no-subpoena games*.

No-subpoena games are realistic extensions of discovery games for two reasons. First, discovery rules apply to both litigants in civil trials, but predominantly to the prosecution in criminal trials.[26] If discovery rules only apply to one litigant (say, P) then E_D must include E_P. A no-subpoena game would then be played if P knew E_D but could not subpoena witnesses in $E_D \backslash E_P$. This situation is exemplified by the 5^{th} Amendment right not to testify in criminal trials: P may know that D is factually guilty, but cannot call D as a witness. Second, discovery rules are sometimes difficult to enforce because a litigant may not be able to adequately describe the requested evidence: for example, D may be sure that P has exonerating evidence in its files, but does not know which file to search. In these circumstances a no-subpoena game would again be played; but E_l need not contain E_m for every evidence set pair in Σ.

5.1.1. The burden of proof

In this part, we provide an example of a no-subpoena game in which litigants might prefer to present first because of our assumption that the leader bears the burden of proof. We also amend this example to provide a related reason (beyond IES) why litigants may ex ante prefer to present second.

Litigants can only pool at evidence set pairs $E \equiv E_D, E_P$ and $F \equiv F_D, F_P$ if J observes the same litigant choices. As the leader must meet a burden of proof, this is only possible if the leader can present the same evidence at E and at F. (The follower could then pass at E and at F.) Reversing the order of presentation could then prevent J from observing the same evidence pair at E and at F. Example 3 below illustrates how this argument could result in litigants preferring to present first.

Example 3 *There are four states: $S = \{1, 2, i, j\}$, where the defendant is factually guilty in the numbered states. There are three witnesses:*

$$e = \{1, i, j\},\ f = \{i, j\} \text{ and } g = \{1, 2, j\}$$

and four evidence sets:

$$E = \{e\},\ F = \{f\},\ G = \{g\} \text{ and } K = \{e, f\}$$

whose conditional distribution (with D's evidence written first) satisfies

$$\pi_1(E, G) = \pi_2(G, G) = \pi_i(E, F) = \pi_j(K, G) = 1.$$

The prior distribution satisfies

$$\frac{p_1}{p_1 + p_i} < d < \min\{\frac{p_1}{p_1 + p_j}, \frac{p_2}{p_2 + p_j}\}. \blacksquare$$

[26] Failure of the prosecution to provide the defendant in a criminal trial with material evidence violates due process (*Brady* v. *Maryland* 373 US 83 (1963)), though these rights were restricted in *Kyles* v. *Whitley* (1995). Under Federal Rules of Criminal Procedure 16, a defense request for discovery triggers a reciprocal obligation to give notice of evidence and witnesses to be called. Litigants in civil cases must disclose evidence, even if it is not requested by the opposing side. (The rules are detailed in Federal Rules of Civil Procedure (2010) Title V and the 1998 Civil Procedure Rules for England.)

Both witnesses in f directly prove factual innocence; so presentation of f induces acquittal.

$\Gamma^{D,P}$ has a separating equilibrium in which J observes e, g at E, G; $g, pass$ at G, G; e, f at E, F; and $ef, pass$ at K, G. The lower bound on d implies that $\Gamma^{D,P}$ also has an equilibrium in which J acquits after observing $e, pass$ at E, G and at E, F; convicts after observing $g, pass$ at G, G; and acquits after observing $ef, pass$ at K, G. On the other hand, the upper bound on d and the availability of f at K preclude an equilibrium in which J observes the same evidence pair at E, G and at K, G.

$\Gamma^{P,D}$ has a separating equilibrium in which J convicts after observing g, e at E, G and $g, pass$ at G, G; and acquits after observing $f, pass$ at E, F, and g, ef at K, G. As F and G have an empty intersection, $\Gamma^{P,D}$ cannot have an equilibrium in which J observes the same evidence pair at E, F and at E, G. The upper bound on d and the availability of f at K preclude an equilibrium in which J observes the same evidence pair at G, G and at K, G.

In sum, $\Gamma^{D,P}$ has a separating equilibrium and another equilibrium with a wrongful acquittal at E, G; while $\Gamma^{P,D}$ only has a separating equilibrium. Consequently, litigants ex ante prefer to present first, while J ex ante prefers P to present first. Example 3 therefore illustrates how the burden of proof may explain why litigants can ex ante prefer to present first in no-subpoena games. Specifically, the burden requires the leader not to pass; so pooling at two evidence set pairs is only possible if the leader has a common witness at those evidence set pairs.

Litigants might also ex ante prefer to present second because of the way the burden of proof determines play in no-subpoena games. To see this, consider Example 3': a variant on Example 3.

Example 3' *There are three states:* $S = \{1, 2, i\}$*, where* i *is the only factually innocent state. There are four witnesses:*

$$e = \{1, i\},\ f = \{i\},\ g = \{2\} \text{ and } h = \{1\}$$

and four evidence sets:

$$E = \{e, f\},\ F = \{h\},\ G = \{e\} \text{ and } H = \{g\},$$

whose conditional distribution (with D's evidence written first) satisfies $\pi_1(F, G) = \pi_2(H, H) = \pi_i(E, E) = 1$.

The prior distribution satisfies $\frac{p_1}{p_1 + p_i} < d$.■

E and F have an empty intersection; so $\Gamma^{D,P}$ can only have a separating equilibrium, in which J only acquits at E, E. $\Gamma^{P,D}$ has a separating equilibrium, in which J convicts after observing $g, pass$ at H, H and either e, h or $e, pass$ at F, G; P presents ef and D completes the full report at E, E because f directly proves innocence (so J acquits). $\Gamma^{P,D}$ also has an equilibrium in which P presents e at E, E and at F, G, to which D responds by passing, while J acquits after observing $e, pass$. P cannot profitably deviate at E, E because witness f directly proves factual innocence, and has no other evidence available at F, G. Comparing this equilibrium with the separating outcome, we see that litigants ex ante prefer to present second.

Examples 3 and 3' exploit the possibility that the evidence available to D at both of two evidence set pairs may differ from the evidence available to P at those evidence set pairs This is impossible in discovery games. Litigants may ex ante prefer to present second in discovery games; but Proposition 2 states that this requires the inter-locking of evidence sets in IES. Example 3' shows that the burden of proof may explain an ex ante preference to present second in no-subpoena games.

5.1.2. No recall

In this part, we first provide an example of a no-subpoena game in which litigants might ex ante prefer to present first because of our assumption that a witness who has already testified cannot be recalled by the follower. The burden of proof reason which we instanced in the last part does not apply here because D has common evidence at some evidence set pairs if and only if P has common evidence at those evidence set pairs. We then amend this example to provide a related reason (beyond IES) why litigants may ex ante prefer to present second.

The no recall assumption is a relevance rule in trials. However, it is also realistic in debates where evidence is presented in public, but which are not governed by procedural rules. Specifically, the fact that some evidence (say, e) is available to a litigant l may itself be informative; and, in some of these cases, l can only demonstrate availability by presenting e first.[27] If litigant l presents second then it can no longer prove availability once its rival has presented e. This is, of course, impossible in discovery games, where J knows that a witness is available to one litigant if and only if it is available to both litigants.

Example 4 *There are five states:* $S = \{1, 2, 3, 4, c\}$*, and the defendant is only factually guilty in state* c*. There are four witnesses:*

$$e = \{1, 2, 3, c\},\ f = \{1, 2, c\}\quad g = \{1, 3, c\}\ \text{and}\ h = \{2, 3, 4\}$$

and four evidence sets:

$$E = \{e, f\},\ F = \{e, g\},\ G = \{f, g\}\ \text{and}\ H = \{h\}$$

whose conditional distribution (with D's evidence written first) satisfies

$$\pi_1(E, G) = \pi_k(H, H) = \pi_c(F, E) = 1 : k = 2, 3, 4.$$

The prior distribution satisfies $\frac{p_c}{p_1 + p_c} < d$.■

[27] In other words, l has no other evidence which directly proves that $e \in E_l$. This condition mirrors the conventional supposition that l cannot prove its ignorance.

D can only present f or h [resp. g] in factually innocent [resp. guilty] states; whereas, P can only present e [resp. g or h] in factually guilty [resp. innocent] states.

$\Gamma^{D,P}$ has a non-separating equilibrium in which D presents e and P responds by passing at E, G and at F, E, and J acquits after observing e, ε: all $\varepsilon \in \{pass, f, g, fg\}$; while D presents h at H, H. P cannot profitably deviate at F, E because it cannot present ef once D has presented e. J would have to observe $f, pass$ or f, e in any such equilibrium of $\Gamma^{P,D}$. However, this is impossible because P could then profitably deviate to presenting e or ef at F, E, as P's presentation of e directly proves factual guilt.

On the other hand, both games have a separating equilibrium. In $\Gamma^{D,P}$, D cannot profitably deviate from presenting ef at E, G, h at H, H and eg at F, E if J convicts after observing e, f and/or $e, pass$. In $\Gamma^{P,D}$, P cannot profitably deviate from presenting fg at E, G, h at H, H and ef at F, E if J acquits after observing f, e and/or $f, pass$.

In sum, $\Gamma^{D,P}$ has a separating and a non-separating equilibrium, while $\Gamma^{P,D}$ only has a separating equilibrium. D is acquitted at H, H in all of these equilibria, but is only acquitted at F, E in the non-separating equilibrium. Consequently, litigants prefer to present first, even though both $E \cap F$ and $E \cap G$ are nonempty. In addition, J prefers P to present first.

The key to this conclusion is that $\Gamma^{D,P}$ alone has a non-separating outcome, which itself follows from our no recall assumption. Specifically, if D presents e then P cannot respond by presenting evidence which includes e, even though the availability of e to P itself directly proves factual guilt.

Example 4 has another interesting property: after Nature has moved, each litigant's evidence set contains a witness who directly proves the state. This property generalizes provability in discovery games. In the last section, we argued that provability precludes any player ex ante preferring an order in discovery games; whereas our analysis of Example 4 demonstrates that litigants might then ex ante prefer an order in no-subpoena games.

Litigants might also ex ante prefer to present second because of the way no recall determines play in no-subpoena games. To see this, consider Example 4': a variant on Example 4:

Example 4' *There are five states:* $S = \{1, 2, 3, 4, c\}$*, and the defendant is only factually guilty in state* c*. There are four witnesses:*

$$e = \{1, 2, 3, c\},\ f = \{1, 2, c\} \quad g = \{1, 3, c\} \text{ and } h = \{2, 3, 4\}$$

and five evidence sets:

$$E = \{e\},\ F = \{f\},\ G = \{g\},\ H = \{h\} \text{ and } K = \{e, f\}$$

whose conditional distribution (with D's evidence written first) satisfies

$$\pi_1(E, K) = \pi_2(K, F) = \pi_3(G, E) = \pi_4(H, H) = \pi_c(E, F) = 1.$$

The prior distribution satisfies

$$\frac{p_c}{p_1 + p_2 + p_c} < d < \min\{\frac{p_c}{p_1 + p_c}, \frac{p_c}{p_2 + p_c}\}. \blacksquare$$

Note that D would prove factual innocence by presenting f and g, and that each litigant would prove factual innocence by presenting h.

$\Gamma^{P,D}$ does not have a separating outcome because P must present f at both E, F and K, F, and J would have to convict at E, F after observing f, e and $f, pass$; so D could not be acquitted at K, F because the no recall assumption precludes it from presenting f. Analogous arguments preclude $\Gamma^{P,D}$ having an equilibrium in which litigants present the same evidence pair at E, F and at E, K (but not at K, F) or an equilibrium in which litigants present the same evidence pair at E, F and at K, F (but not at E, K): J would again have to convict after observing $f, \varepsilon : \varepsilon \in \{e, pass\}$; so D could not be acquitted at K, F. The only equilibrium in $\Gamma^{P,D}$ prescribes P to present f at E, F, at K, F and at E, K; D to pass at each of these evidence sets pairs; and J to acquit after observing $e, pass$, $f, pass$, $h, pass$ and $ef, pass$.

$\Gamma^{D,P}$ does not have a separating outcome because D must present e at both E, F and E, K, and J would have to acquit at E, K after observing e, f and $e, pass$; and D would then be acquitted at E, F. $\Gamma^{D,P}$ cannot have an equilibrium which prescribes conviction at K, F because D's presentation of f would induce acquittal; so J cannot observe the same evidence pair at E, F and at K, F. On the other hand, $\Gamma^{D,P}$ has an equilibrium which prescribes D to present e at E, F and at E, K, f at K, F, g at G, E, and h at H, H; P to respond by passing at every evidence set pair; and J to convict after observing $e, pass$ and to acquit after observing $f, pass$, $g, pass$, g, e and $h, pass$. J wrongfully convicts at E, K in this equilibrium. Finally, $\Gamma^{D,P}$ also has an equilibrium which prescribes D to present e at E, F, at E, K and at K, F, g at G, E, and h at H, H; P to respond by passing at every evidence set pair; and J to acquit after observing $e, pass$, e, f, $g, pass$, g, e and $h, pass$.

In sum, $\Gamma^{P,D}$ only has an equilibrium with a wrongful acquittal at E, F, while $\Gamma^{D,P}$ also has an equilibrium with a wrongful conviction at E, K. We therefore conclude that litigants ex ante prefer to present second in Example 4'. This preference turns on our no recall assumption rather than on IES. Specifically, the equilibrium in which J observes the same evidence pair at E, F and at E, K alone cannot exist in $\Gamma^{P,D}$ because P would then secure a conviction at K, F by presenting f, which prevents D from proving its factual innocence by presenting f. In contrast to our previous examples, J does not ex ante prefer an order in Example 4' because it does not prefer one equilibrium outcome over the other in $\Gamma^{D,P}$: one equilibrium prescribes a wrongful conviction at E, K; the other equilibrium prescribes a wrongful acquittal at E, F.

Example 4' demonstrates two more general features of no-subpoena games: that the equilibrium outcome correspondences of $\Gamma^{D,P}$ and $\Gamma^{P,D}$ might be disjoint; and that neither game might have a separating equilibrium. By contrast, Proposition 1 states that both discovery games have a separating equilibrium.

Comparing Examples 4 and 4', the no recall assumption results in an ex ante preference to present first (Example 4) by preventing the follower from presenting direct evidence in its favor on the equilibrium path; whereas the assumption results in an ex ante preference to present second (Example 4') by rendering profitable some deviation by the leader from a putative equilibrium path which prevents the follower from presenting direct evidence in the follower's favor.

5.2. Incomplete information games

In the last subsection, we noted that asymmetric or unenforceable discovery rules may result in the play of no-subpoena games, provided that each litigant knows the evidence available to its rival. In this subsection, we drop the latter condition. Specifically, we consider games in which each litigant may be uncertain of its rival's available evidence. We will call such cases *incomplete information games.*

We start with a variant on Example 1 (in Section 3.3), which illustrates how a litigant may ex ante prefer to present first in such games.

Example 5 *There are three states:* $S = \{m, o, mo\}$*, where* mo *is the only factually guilty state. There are two witnesses:*

$$e = \{m, mo\} \text{ and } f = \{o, mo\}$$

and three evidence sets:

$$M = \{e\},\ O = \{f\} \text{ and } MO = \{e, f\},$$

whose conditional distribution (with D's evidence written first) satisfies

$$\pi_m(M, M) = \pi_o(O, O) = 1;\ \pi_{mo}(MO, M) = 1 - \pi_{mo}(MO, O) \equiv \pi \in (\frac{1}{2}, 1).$$

The prior distribution satisfies

$$\frac{p_{mo}\pi}{p_{mo}\pi + p_m} < d < \frac{p_{mo}(1-\pi)}{p_{mo}(1-\pi) + p_o}.\blacksquare$$

The interpretation of m and o as motive and opportunity in Example 1 carries over to Example 5.

As $\pi > 1/2$, the following strategy combination is an equilibrium in $\Gamma^{D,P}$:

- D presents e when its evidence set is M or MO, and otherwise presents f;
- P completes the full report at all of its evidence sets;
- J acquits after and only after observing either $e, pass$ or $f, pass$.

This equilibrium is unique: for D cannot present ef when its evidence set is MO because J would then acquit after observing $e, pass$ and $f, pass$; so D could profitably deviate to presenting either e or f. Furthermore, D cannot present f when its evidence set is MO because J would then acquit after observing $e, pass$; so $\pi > 1/2$ implies that D could profitably deviate to presenting e.

The following strategy combination is an equilibrium in $\Gamma^{P,D}$:

- P presents e when its evidence set is M, and otherwise presents f;

- D always passes;
- J acquits after and only after observing $e, pass$.

In every equilibrium, D passes in response to P presenting e [resp. f]: else J would then convict, and would acquit after observing $e, pass$ [resp. $f, pass$]; so D could profitably deviate to passing. Consequently, this game has a unique equilibrium outcome.

Comparing equilibrium outcomes in the two games: each litigant ex ante prefers to present first because J reaches the same verdict in both games at every evidence set pair other than O, O, where J only acquits in $\Gamma^{D,P}$.[28]

This result is rather striking in light of (our reading of) a claim by Damaska (1973) which we quoted in Section 2: that incomplete information provides an additional reason for preferring to present second because the follower can then not be surprised by the leader's available evidence. Learning that P has witness f at MO, O does not allow D to secure an acquittal; but its equilibrium response at MO, O harms D at O, O.

Finally, note that J ex ante prefers an order, even though neither $\Gamma^{D,P}$ nor $\Gamma^{P,D}$ have a separating equilibrium: J ex ante prefers D to present first because D is then rightfully acquitted at O, O.

A simple variant on Example 5 reveals that litigants may again ex ante prefer to present second in incomplete information games for reasons which are unrelated to IES.

Example 5' *As Example 5, except that the prior distribution satisfies*

$$\max\{\frac{p_{mo}\pi}{p_{mo}\pi + p_m}, \frac{p_{mo}(1-\pi)}{p_{mo}(1-\pi) + p_o}\} < d. \blacksquare$$

Each game has a unique equilibrium. Litigant play corresponds to that in Example 5; and J reaches the same verdict as in Example 5 at each evidence set pair when D presents first. The difference concerns the equilibrium verdicts in $\Gamma^{P,D}$, where J now acquits at O, O and at MO, O; so J acquits at every evidence set pair in equilibrium. Litigants then prefer to present second, even though every miscarriage of justice is a wrongful acquittal in the only equilibrium of $\Gamma^{D,P}$.

We argued above that Example 3 [resp. 3'] illustrates the burden of proof reason why litigants may ex ante prefer to present first [resp. second] in a no-subpoena game. Examples 5 and 5' share this property: e is available at evidence sets M and MO; and $\Gamma^{D,P}$ has an equilibrium in which D presents e at evidence set pairs M, M, MO, M and

[28] Arguments analogous to those used above imply that litigants would also prefer to present first in a variant on Example 5 in which

$$\frac{p_{mo}(1-\pi)}{p_{mo}(1-\pi) + p_o} < d < \frac{p_{mo}\pi}{p_{mo}\pi + p_m} \text{ and } \pi < \frac{1}{2}.$$

MO, O. However, this property is not sufficient for litigants to present the same evidence pairs at MO, M and at M, M or at MO, O and at O, O.

We summarize this section's main results in

Theorem 2 *Litigants may prefer to present first in non-discovery games.*■

6. Commitment

In the last section, we provided three examples of games in which litigants ex ante prefer to present first. In this section, we will argue that these examples share a common feature: the follower's ability to respond on a case-by-case basis may impose negative externalities across cases (where 'cases' are available evidence set pairs) which would be resolved if the follower could commit ex ante to its strategy. We will argue that, in these examples, games in which the follower can commit have the same outcomes as games in which the order of presentation is reversed. In other words, presenting first acts like a commitment device.

Negative externalities do not rely on there being two litigants; so, for simplicity, suppose it to be commonly known that the leader has a single witness available at each evidence set pair, while the follower has several available witnesses at some evidence set pairs (so the game is no-subpoena).

Suppose that Σ contains evidence set pairs E and F such that $v(E) = v(E \cup F) = v_1 \neq v(F)$. If $E_1 = F_1$ and $F_2 \subset E_2$ then J cannot reach different verdicts at E and at F because the follower would exercise its flexibility by presenting the same evidence at E as at F. However, J would reach v_2 at F if the follower could commit to presenting some evidence in $E \backslash F$ at E. A related argument demonstrates that commitment could allow litigants to present the same evidence pair at different evidence set pairs. Specifically, suppose that Σ contains evidence set pairs E, F and G such that $v(E) = v(G) = v(E \cup F \cup G) = v_1$ and $v(F) = v(E \cup F) = v_2$. If $E_1 = F_1 = G_1$ and $E_2 \cap F_2 \subset G_2$ then J cannot reach different verdicts at $E \cup F$ and at G because the follower would exercise its flexibility by presenting the same evidence at G as at $E \cup F$. However, J would reach v_2 at $E \cup F$ if the follower could commit to presenting some evidence in $G_2 \backslash E_2 \cap F_2$ at G.

We formalize these ideas by defining a commitment game. We start with a given order of presentation, identifying one of the litigants as the follower. We then define a game in which the follower commits to its strategy before Nature moves. Commitment games of this sort have been studied by Chen and Olszewski (2014), who interpret commitment as reputation. Litigants are typically not repeat players; so we think of commitment games as hypothetical constructs which elucidate the idea that flexibility might be so detrimental that litigants ex ante prefer to present first.

Take some game $\Gamma^{1,2}$. The commitment game $\Gamma_c^{1,2}$ has the following time line. The follower starts play by choosing its strategy: that is, the evidence it presents in response to the leader's presentation at each follower evidence set. Nature then selects the state and the evidence set pair; the leader presents evidence, having observed its own evidence set and the follower's strategy; the follower implements its chosen strategy; and J reaches a verdict after observing the follower's strategy and the evidence pair. Players have the same payoff functions as in $\Gamma^{1,2}$. It is important, for our purposes, that J observes the follower's commitment: cf. Remark at the end of this section.

$\Gamma_c^{1,2}$ has Bayes perfect equilibria where J punishes the follower for deviating to another strategy by reaching v_1 whenever the evidence pair is ambiguous. This punishment seems implausible because the follower moves before observing its available evidence (so the strategy combination would not be a sequential equilibrium).[29] Specifically, let $e_2(e)$ denote the follower's strategy. On any path, J observes some $e_2(e)$ and an evidence pair e_1, e_2. As $e_2(e)$ is a commitment, J can infer the evidence set pairs E_1, E_2 at which litigants could have presented e_1, e_2. We focus on strategy combinations at which J's belief about E_1, E_2 only depends on $e_2(e)$ via this effect. In this section, we will abuse terminology by referring to pure strategy Bayes perfect equilibria which satisfy this property and at which the leader does not pass as the *equilibria* of $\Gamma_c^{1,2}$. We denote the outcome correspondence of $\Gamma_c^{1,2}$ by $w_c^{1,2}$.

We now show that $w_c^{1,2} = w^{2,1}$ for some $1 \in \{D, P\}$ in Examples 3, 4 and 5:

Example 3

In this example, $\Gamma^{D,P}$ has a separating equilibrium and an equilibrium in which J wrongfully acquits at E, G. By contrast, $\Gamma_c^{D,P}$ has a unique equilibrium in which P commits to presenting f at E, F and passing at E, G; so J convicts at E, G and G, G, and acquits at E, F and K, G, which is the separating outcome. This is the only equilibrium of $\Gamma_c^{D,P}$, as P could otherwise profitably deviate to its strategy in the separating equilibrium. On the other hand, $\Gamma^{P,D}$ only has a separating equilibrium. In sum, presenting first replicates the effect of committing ex ante.

Example 4

In this example, $\Gamma^{D,P}$ has an equilibrium in which J acquits after observing $e, pass$ at E, G and at F, E, and a separating equilibrium in which D directly proves innocence [resp. guilt] by presenting f [resp. g] at E, G [resp. F, E]. $\Gamma_c^{D,P}$ cannot have an equilibrium with a wrongful acquittal at F, E. To see this suppose otherwise, and consider a deviation by P to presenting f in response to e at F, E and passing in response to e at E, G. If D presented e at F, E then J would observe e, f and would have to convict, as e, f would only be played at F, E after P's (observed) deviation; and if D presented g or eg then J would again convict. P's deviation would therefore be profitable. On the other hand, $\Gamma_c^{D,P}$ has a separating equilibrium.

Every equilibrium in $\Gamma^{P,D}$ is separating: a property shared with $\Gamma_c^{D,P}$. In $\Gamma^{P,D}$, D cannot prevent P from directly proving guilt; in $\Gamma_c^{D,P}$, P's commitment ensures that J can identify E, G, irrespective of the evidence that D presents there. Presenting first therefore replicates the effect of an ex ante commitment.

Example 5

In this example, $\Gamma^{P,D}$ has a unique equilibrium in which D always passes, and is wrongfully convicted at O, O. $\Gamma_c^{P,D}$ has a unique equilibrium in which D commits to presenting e at MO, O and otherwise passing. J then acquits after observing $e, pass$ and $f, pass$, and therefore rightfully acquits at O, O. This equilibrium is unique because D

[29] Fudenberg and Tirole (1991) p.332 build this condition ("no signaling what you don't know") into their definition of a Bayes perfect equilibrium.

could otherwise profitably deviate to presenting e at MO, O and otherwise passing (viz. to its equilibrium strategy).

$\Gamma^{D,P}$ has an equilibrium in which J rightfully acquits at O, O because, by presenting first, D is forced to present the same evidence at MO, M and at MO, O. This replicates the equilibrium of $\Gamma_c^{P,D}$, where D commits to presenting the same evidence at the two evidence set pairs.

In sum, $w^{2,1}$ and $w_c^{1,2}$ coincide in Examples 3, 4 and 5 for *an* assignment of roles: viz. some $1 \in \{D, P\}$. In Example 3, this is true for both assignments: each litigant as follower prefers to commit, and the equilibrium outcomes in each pair ($\Gamma_c^{D,P}$,$\Gamma^{P,D}$) and ($\Gamma_c^{P,D}$,$\Gamma^{D,P}$) are the same. However, commitment is not valuable for D in Example 4 or for P in Example 5.

In Examples 3, 4 and 5, a commitment game $\Gamma_c^{1,2}$ has the same outcome(s) as $\Gamma^{2,1}$. This property does not generalize to all no-subpoena games. In Example 6 below, we demonstrate that $\Gamma_c^{1,2}$ may have a unique outcome which the follower prefers to any outcome in either $\Gamma^{1,2}$ or $\Gamma^{2,1}$.

Example 6 *There are five states:* $S = \{1, 2, 3, c_1, c_2\}$*, where* c_1 *and* c_2 *are the only factually guilty states. There are three witnesses:*

$$e = \{1, 2, c_1, c_2\},\ f = \{1, 2, 3, c_1\} \text{ and } g = \{2, 3, c_1, c_2\}$$

and six evidence sets:

$$E = \{e\},\ F = \{f\},\ G = \{g\},\ H = \{e, f\},\ K = \{e, g\} \text{ and } L = \{f, g\}$$

whose conditional distribution (with D's evidence written first) satisfies

$$\pi_1(E, F) = \pi_2(E, L) = \pi_3(F, F) = \pi_{c_1}(K, H) = \pi_{c_2}(G, E) = 1.$$

The prior distribution satisfies

$$\max\{\frac{p_{c_1}}{p_{c_1} + p_2}, \frac{p_{c_1}}{p_{c_1} + p_3}\} < d < \frac{p_{c_1}}{p_{c_1} + p_1}.\blacksquare$$

Note that D presenting g induces conviction; while P presenting g [resp. e] induces acquittal [resp. conviction].

$\Gamma^{D,P}$ cannot have a separating equilibrium, as J would then have to acquit at E, F and at E, L; so D could profitably deviate to presenting e at K, H. It can also not have an equilibrium in which J convicts after observing the same evidence pair, $e, pass$ or e, f, at K, H and at E, F and observes e, g or e, fg at E, L: for P could then profitably deviate to presenting f or passing at E, L. However, it has equilibria in which J acquits after observing $e, pass$ [resp. e, f] at K, H and at E, L and e, f [resp. $e, pass$] at E, F. These equilibria prescribe a wrongful acquittal at K, H.

$\Gamma^{P,D}$ has a separating equilibrium in which J observes $ef, pass$ at K, H, f, $pass$ at E, F, E, L, and F, F, and e, g at G, E. Indeed, the condition on priors implies that J must acquit at F, F in any equilibrium; and, as e is available to P at K, H, J must convict at K, H. Finally, $\Gamma^{P,D}$ cannot have an equilibrium in which J observes either $f, pass$ or f, e at both K, H and at E, F and another evidence pair at E, L: for it would then convict at K, H and at E, F and acquit at E, L; so P could profitably deviate at E, L.

In sum, neither $\Gamma^{D,P}$ nor $\Gamma^{P,D}$ have an equilibrium in which J convicts at E, F. By contrast, J convicts at E, F in every equilibrium of $\Gamma_c^{D,P}$. To see this, note that $\Gamma_c^{D,P}$ has an equilibrium in which P commits to pass in response to e at K, H and at E, F and to present f in response to e at E, L. The outcome is unique because P could otherwise profitably deviate to passing in response to e at K, H and at E, F and to presenting f in response to e at E, L.

Our analysis thus far has demonstrated that a follower may prefer to commit in non-discovery games, and that $w_c^{1,2}$ and $w^{2,1}$ may then coincide. We end this section by applying our analysis to discovery games.

Example 1 (in Section 3.3) illustrates a situation in which the follower ex ante prefers to commit in a discovery game. Absent commitment, both games have a separating equilibrium and an equilibrium in which the only miscarriage of justice is a wrongful conviction. D completes the full report at every equilibrium of $\Gamma_c^{P,D}$, yielding the separating outcome. Consequently, D prefers to commit, as in Examples 3, 4 and 5.[30]

In Example 1, $\Gamma^{2,1}$ and $\Gamma_c^{1,2}$ both have separating equilibria. Proposition 1 states that all discovery games have a separating equilibrium. Proposition 3 below asserts that this property also holds when the follower can commit:

Proposition 3 *Every commitment game which is played under discovery has a separating equilibrium.*

Proof We need some notation before defining the requisite strategy combination in $\Gamma_c^{1,2}$. Suppose that the follower has committed to present $e_2^E(e)$ in response to evidence e at evidence set E, where $e_2^E(e) = pass$ if e is the full report at E. For each $e \in E$, let $E(e)$ denote the evidence sets at which the follower commits to present $e_2^E(e)$ in response to evidence e.

Consider the following strategy combination in $\Gamma_c^{1,2}$. The follower commits to completing the full report at every evidence set; the leader presents the full report at every evidence set; and J reaches verdict $v(E)$ if e_2 completes the full report at some evidence set E, and otherwise reaches v_2 unless $e_1 e_2$ induces verdict v_1 or $v(E) = v_1$ for every $E \in E(e)$. We claim that this strategy combination plus the following beliefs form an equilibrium in $\Gamma_c^{1,2}$:

After observing e_1, e_2, J believes that the realized evidence set is: E if $e_1 e_2$ is the full report at E; is in Σ_α [resp. Σ_γ] if $e_1 e_2$ induces acquittal [resp. conviction]; and is otherwise in Σ_{v_2}.

[30] It is easy to confirm that the *leader* may prefer to commit in Example 1: in $\Gamma^{P,D}$, P could commit to presenting f whenever it is available, excluding the separating equilibrium of $\Gamma^{P,D}$; in $\Gamma^{D,P}$, D could commit to presenting the full report, excluding the non-separating equilibrium of $\Gamma^{D,P}$.

Given litigant strategies, J cannot profitably deviate when e_2 completes the full report at E. J can also not profitably deviate if $e_1 e_2$ induces a verdict or if $v(E) = v_1$ for every $E \in E(e)$. The evidence pair is otherwise ambiguous, so J cannot profitably deviate from reaching verdict v_2.

The leader's strategy ensures that it secures v_1 at every evidence set $E \in \Sigma_{v_1}$. If $E \in \Sigma_{v_2}$ then the leader has no evidence $e_1 \in E$ such that $e_1, e_2^E(e_1)$ induces v_1. Furthermore, $v(E) = v_2$ implies that, for every $e \in E$: $E(e) \cap \Sigma_{v_2}$ is non-empty. Consequently, the leader cannot profitably deviate at any evidence set.

The follower anticipates that J will reach $v(E)$ at evidence set E, irrespective of its commitment; so it cannot profitably deviate from committing to complete the full report.■

Our analysis of discovery games reveals that the follower may prefer to commit, and that $\Gamma_c^{1,2}$ and $\Gamma^{2,1}$ both have separating equilibria. One might therefore conjecture that the properties of Examples 3, 4 and 5 may carry over to discovery games: viz. that litigants might ex ante prefer to present first because $w_c^{1,2} = w^{2,1}$. Lemma precludes this possibility. If $x \succ_2 y$ for some $x \in w^{1,2}$ and $y \in w^{2,1}$ then $w^{1,2}$ and $w^{2,1}$ respectively contain y' and x' such that $y' \succeq_2 x$ and $y \succeq_2 x'$, contrary to Condition 1 (ex ante). By contrast, this condition is always satisfied in $\Gamma_c^{1,2}$ because the follower can commit to its strategy in any equilibrium of $\Gamma^{1,2}$ (irrespective of whether litigants share available evidence).[31]

Remark Our analysis of Examples 3, 4 and 5 relies on our assumption that both the leader and J observe the follower's strategy in $\Gamma_c^{1,2}$. Suppose, per contra, that the leader alone observes the follower's commitment in Example 3. This variant on $\Gamma_c^{D,P}$ has an equilibrium in which P commits to passing in response to e at H, E and at E, F, replicating the equilibrium outcome in $\Gamma^{D,P}$: for P must pass in response to e at H, E, while e, f induces an acquittal; so a deviation by P to presenting f in response to e at E, F would not change the verdict that J reaches at any evidence set, and is therefore unprofitable. This variant on $\Gamma_c^{D,P}$ also has a separating equilibrium (in which P commits to present f at E, F); so its equilibrium outcomes do not coincide with $w^{P,D}$ (which consists of the separating outcome).

7. Ex post preferences

Thus far, we have considered when litigants can prefer an order of presentation before knowing their available evidence. This perspective is relevant to the design of procedural rules for future trials. In some non-judicial contexts, a 'litigant' might choose the presentation order before observing its available evidence: for example, a committee chair might choose a procedure to govern all future committee meetings.[32] In this section we take an ex post approach to studying preference over the order of presentation. Specifically, we consider the order which a given litigant (say, D) would choose *after observing its available evidence*. This perspective is relevant in Italian criminal trials, where the defendant may ask to present first: defendants typically arrive at trial having observed their available evidence, and are not repeat players. It is also relevant in some non-judicial contexts, such as

[31] The follower can therefore not ex ante prefer not to commit.

[32] The 'verdict' would then be interpreted as the committee's post-deliberation decision.

committees, whose chair may decide the order in which members speak on a case-by-case basis.

The game which we analyze in this section starts with Nature choosing the state and available evidence sets, revealing the latter to the litigants; D then chooses an order, and the ensuing game is then played out according to the rules specified in Section 3. We focus on games in which litigants share available evidence, as in the discovery games of Section 4. We denote such games by Γ, and refer to them as ex post order games. We analyze Γ by characterizing its equilibria, as defined in Section 3.

Our first result compares outcomes in ex post order games with outcomes in the discovery games of Section 4.

Proposition 4 *Every outcome in a discovery game ($\Gamma^{D,P}$ or $\Gamma^{P,D}$) is an outcome in the ex post order game (Γ).*

Proof Let X denote an equilibrium of a discovery game with a given order, which we denoted $\Gamma^{l,m}$, where $l \neq m \in \{D, P\}$. We will argue that Γ has an equilibrium in which D chooses order l, m at every evidence set, and X is then played.

By definition of X, no player can profitably deviate once D has chosen order l, m: J's belief that this choice of an order does not inform J about the realized evidence set is sequentially rational because D chooses this order at every evidence set.

Suppose that D deviates to choosing order m, l, and consider the following strategy combination in the continuation:

- The leader (litigant m) presents the full report at every evidence set;
- If D is the follower then it completes the full report; if P is the follower then it passes;
- J acquits after observing evidence pair e, f if ef induces acquittal, and otherwise convicts.

After observing the unexpected order m, l and some evidence pair e_1, e_2, J believes that the realized evidence set is in Σ_α if and only if $e_1 e_2$ induces acquittal. These beliefs satisfy feasibility (cf. Section 3.2). Given these beliefs, J can never profitably deviate after observing the unexpected order.

If P is the follower and D has presented evidence which induces acquittal then J would acquit, irrespective of the evidence which P presents; so P cannot profitably deviate from passing. P can also not profitably deviate if D has presented evidence which does not induce acquittal, as J then convicts. If D is the follower then it cannot profitably deviate because the full report induces acquittal if any evidence contained in the full report induces acquittal.

D as leader cannot profitably deviate: again because the full report induces acquittal if any evidence contained in the full report induces acquittal. P as leader cannot profitably deviate because it expects D to complete the full report.

We now turn to D's choice of an order. As X is an equilibrium of $\Gamma^{l,m}$, it must prescribe acquittal at every evidence set whose full report induces acquittal. The specified strategy combination implies that J would only acquit at those evidence sets if D deviated to order m, l. Consequently, D cannot profitably deviate to choosing order m, l at any evidence set. The result follows from the definition of X, where J's beliefs after D chooses the expected order correspond to its beliefs in the equilibrium of $\Gamma^{l,m}$, and therefore satisfy Bayes rule.■

Proposition 4 implies that D chooses each given order at every evidence set in an equilibrium of Γ, even though it never ex ante prefers to present first (cf. Theorem 1a)). In one of these equilibria, D is deterred from presenting second at any evidence set because J draws the most skeptical possible inference from order P, D and any evidence pair. Propositions 1 and 4 jointly imply that Γ has a separating equilibrium.

We now use Example 7 (below) to demonstrate that ex post order games may have outcomes which are not outcomes in either discovery game. Example 7 extends Example 2, which we provided in the proof of Theorem 1b) to illustrate how $\Gamma^{P,D}$ could alone have an equilibrium with a wrongful acquittal. Example 7 adds four further states and four further evidence sets which mirror those in Example 2.

Our analysis of Example 7 will also demonstrate that Γ has equilibria in which D chooses order D, P at some, but not all evidence sets. By contrast, D chooses the same order at every evidence set in the equilibrium constructed to prove Proposition 4.

Example 7 *There are eight states:* $S = \{1, 2, 3, 4, i, j, k, l\}$, *where* D *is factually guilty in the numbered states. There are six witnesses:*

$$\begin{aligned} e &= \{1, 2, i\},\ f = \{2, 3, i\},\ g = \{i\}, \\ h &= \{j, k, 4\},\ m = \{k, l, 4\} \text{ and } n = \{4\} \end{aligned}$$

and eight evidence sets:

$$\begin{aligned} E &= \{e, f, g\},\ F = \{e, f\},\ G = \{f\},\ H = \{e\} \\ K &= \{h, m, n\},\ L = \{h, m\},\ M = \{m\},\ N = \{h\} \end{aligned}$$

whose conditional distribution satisfies

$$\pi_1(H) = \pi_2(F) = \pi_3(G) = \pi_i(E) = \pi_j(N) = \pi_k(L) = \pi_l(M) = \pi_4(K) = 1.$$

The prior distribution satisfies

$$\max\{\frac{p_1}{p_1 + p_i}, \frac{p_4}{p_4 + p_k}, \frac{p_4}{p_4 + p_l}\} < d < \min\{\frac{p_2}{p_2 + p_i}, \frac{p_3}{p_3 + p_i}, \frac{p_4}{p_4 + p_j}\}.■$$

All evidence which includes witness g [resp. n] induces acquittal [resp. conviction] in Example 7; all other evidence is ambiguous.

Discovery game $\Gamma^{P,D}$ with the parameters in Example 7 has an equilibrium in which J observes $e, pass$ at E and at H and the full report at every other evidence set; so the only miscarriage of justice is a wrongful acquittal at H. J cannot wrongfully convict at N after observing $h, pass$ because P could then profitably deviate to presenting h at L. (This argument is equivalent to that used in the proof of Claim 2.) The only other outcome of $\Gamma^{P,D}$ is separating. Similarly, the only non-separating outcome of discovery game $\Gamma^{D,P}$ has a wrongful conviction at N.

Now consider the ex post order game Γ with the parameters in Example 7. Proposition 4 implies that Γ has a separating outcome and two other outcomes, each with a single miscarriage of justice. We claim that Γ has another outcome with a wrongful acquittal at H and a wrongful conviction at N. Specifically, consider the following strategy combination and beliefs:

- D chooses order D, P [resp. P, D] at every evidence set in $\Sigma^{D,P}$ [resp. $\Sigma^{P,D}$], where $E \in \Sigma^{P,D}$ and $K \in \Sigma^{D,P}$.

- Consider any evidence set in $\Sigma^{P,D}$. If D has chosen order P, D then P presents the full report at every evidence set except E, where it presents e. D completes the full report unless P has presented e, in which case D passes. If D has chosen order D, P then D presents the full report, and P always passes.

  Consider any evidence set in $\Sigma^{D,P}$. If D has chosen order D, P then D presents the full report at every evidence set except K, where it presents h. P completes the full report unless D has presented h, in which case P passes. If D has chosen order P, D then P presents the full report, and D always completes the full report.

- If a litigant has presented evidence which contains g [resp. n] then J believes that the realized evidence set is E [resp. K] and acquits [resp. convicts].

  If D has chosen order P, D then the table below describes J's belief about the realized evidence set and its verdict after observing any evidence pair e_P, e_D such that $e_P e_D \in \Sigma^{P,D}$ but does not contain g:

Evidence pairs	J's belief	Verdict
$e, pass$	E or H	α
$f, pass$	G	γ
$ef, pass$ or e, f or f, e	F	γ

  If J observes an evidence pair e_P, e_D such that $e_P e_D$ is contained in an evidence set (say, Q) $\in \Sigma^{D,P}$ then J believes that the realized evidence set is K and convicts.

  If D has chosen order D, P then the table below describes J's belief about the realized evidence set and its verdict after observing any evidence pair e_P, e_D such that $e_P e_D \in Q \in \Sigma^{D,P}$ but does not contain n:

Evidence pairs	J's belief	Verdict
$h, pass$	K or N	γ
$m, pass$	M	α
$hm, pass$ or h, m or m, h	L	α

If J observes an evidence pair e_P, e_D such that $e_P e_D \in Q \in \Sigma^{P,D}$ but does not contain g then J believes that the realized evidence set is F and convicts.

J's beliefs after observing any evidence pair satisfy Bayes rule on the path, and are feasible off the path; and J's verdict is sequentially rational. Moreover, J knows when D has deviated to choosing the extra-equilibrium order because evidence sets in $\Sigma^{P,D}$ and $\Sigma^{D,P}$ do not share any common witnesses. Given an order, neither litigant can profitably deviate; and D cannot profitably deviate to choosing an unexpected order.

These arguments imply that the strategy combination and beliefs above are an equilibrium of Γ, where J wrongfully acquits at H and wrongfully convicts at N. By contrast, the two discovery games each feature a single miscarriage of justice. Hence, ex post order games may have outcomes which are neither in $w^{P,D}$ nor in $w^{D,P}$.

The condition on priors in Example 7 implies that Γ has no other equilibrium outcomes. D cannot ex ante prefer to play $\Gamma^{P,D}$ over playing Γ because both games have a separating equilibrium and an equilibrium with a wrongful acquittal at H. However, D ex post prefers to play $\Gamma^{P,D}$ over playing Γ at evidence set N because it is acquitted in every equilibrium in the former case, and convicted in some equilibria otherwise.

Theorem 3 below states that the ex ante property is more general than Example 7.

Theorem 3 *If litigants have the same available evidence then D cannot ex ante prefer playing the ex post order game to playing $\Gamma^{P,D}$, where it always presents second.*

Proof In light of Proposition 4, this result is obvious when $\Gamma^{P,D}$ has a non-separating equilibrium; so suppose otherwise. D then ex ante prefers to play the ex post order game if and only if that game has an equilibrium (say, X) which prescribes some wrongful acquittals and no wrongful convictions. We will prove the result by using X to construct a non-separating equilibrium (say, Y) of $\Gamma^{P,D}$ with the same outcome as X.

X prescribes D to choose order D, P at some evidence sets (say, $\Sigma^{D,P}$), and to choose order P, D at the other evidence sets (say, $\Sigma^{P,D}$). Write $\Sigma_v^{l,m}$ for the evidence sets in $\Sigma^{l,m}$ at which X prescribes verdict v, and $e_l(E)$ for the evidence which X prescribes l to present at E.

Y prescribes

- P to present $e_D(E)$ at every $E \in \Sigma_\alpha^{D,P}$, to present $e_P(E)$ at every $E \in \Sigma_\alpha^{P,D}$, and to present the full report at every other evidence set;
- D to respond to e_P by passing when X prescribes P to present e_P at some evidence set in $\Sigma_\alpha^{D,P} \cup \Sigma_\alpha^{P,D}$, and otherwise to complete the full report;
- J to acquit after observing any evidence e, f (where f may be *pass*) such that ef is in some evidence set in $\Sigma_\alpha^{D,P} \cup \Sigma_\alpha^{P,D}$ unless ef is the full report at some evidence set in $\Sigma_\gamma^{D,P} \cup \Sigma_\gamma^{P,D}$ or is not contained in $\Sigma_\alpha^{D,P} \cup \Sigma_\alpha^{P,D}$.[33]

[33] We write $ef = e$ when $f = pass$.

Y is well-defined because the full report at any singleton evidence set in $\Sigma_\alpha^{D,P} \cup \Sigma_\alpha^{P,D}$ cannot be the full report at any other evidence set; and because no singleton evidence set $F \in \Sigma_\gamma^{D,P} \cup \Sigma_\gamma^{P,D}$ can consist of either $e_D(E)$ for any $E \in \Sigma_\alpha^{D,P}$ or of $e_P(E)$ for any $E \in \Sigma_\alpha^{P,D}$ as, in each case, D could profitably deviate from X by choosing the other order at F.

J cannot profitably deviate because $\Sigma_\gamma^{D,P} \cup \Sigma_\gamma^{P,D} \subseteq \Sigma_\gamma$ as X does not prescribe any wrongful convictions; because it prescribes llitigants to present the same evidence pair at the same collections of evidence sets as X unless $e_D(E) = e_P(F)$ for some $E \in \Sigma_\alpha^{D,P}$ and $F \in \Sigma_\alpha^{P,D}$, in which case J cannot improve on acquitting; and because the full report at any evidence set in $[\Sigma_\alpha^{D,P} \cup \Sigma_\alpha^{P,D}] \cap \Sigma_\gamma$ must be ambiguous, else P could profitably deviate from X by presenting or completing the full report.

Y only prescribes J to convict after observing e, f if ef is the full report at some evidence set in Σ_γ or is not contained in any evidence set in $\Sigma_\alpha^{D,P} \cup \Sigma_\alpha^{P,D}$. Hence, D cannot profitably deviate at any evidence set in $\Sigma_\gamma^{D,P} \cup \Sigma_\gamma^{P,D}$, and therefore at any evidence set.

If $E \in \Sigma_\alpha^{D,P} \cup \Sigma_\alpha^{P,D}$ then Y prescribes J to acquit after observing every e_P, e_D such that $e_P e_D$ is the full report at E; so P cannot profitably deviate at any evidence set in $\Sigma_\alpha^{D,P} \cup \Sigma_\alpha^{P,D}$, and therefore at any evidence set.

These arguments imply that Y is a non-separating equilibrium of $\Gamma^{P,D}$; and D ex ante prefers Y's outcome over the separating outcome. Proposition 4 then implies the result.■

Theorem 3 states that D cannot ex ante prefer to play the ex post order game over always presenting second. D may, however, ex ante prefer to play Γ over always presenting first. Example 2 illustrates this possibility: Claim 2 asserts that $\Gamma^{D,P}$ only has a separating equilibrium; Proposition 4 implies that Γ also has the equilibrium of $\Gamma^{P,D}$ with a wrongful acquittal (cf. Claim 1); and it is easy to confirm that the ex post order game has no other equilibria.

In Example 2, D cannot prefer to play $\Gamma^{P,D}$ over playing Γ because the former game has two equilibrium outcomes. This is not a general property. To see this, consider the mirror image of Example 2 that we built into Example 7. $\Gamma^{P,D}$ then only has a separating equilibrium; $\Gamma^{D,P}$ also has an equilibrium with a wrongful conviction; and the equilibrium outcomes of Γ coincide **with** the outcomes of $\Gamma^{D,P}$. Consequently, D prefers to always present second over playing the ex post order game.

8. Summary and extensions

We have studied litigants' preferences over the order of presentation in a model that tries to capture essential features of common law trials. Our results indicate the importance of distinguishing between games played under discovery and games in which litigants may have different available evidence. In the former case, litigants cannot prefer to present first, but may prefer to present second; in the latter case, litigants might prefer to present first. These results suggest that the objectives of procedure (whatever those are) might better be served by sometimes changing the existing order. Any such change would doubtless require further procedural reforms: for example, requiring that the indictment be accompanied by a more detailed summary of P's case to satisfy the 6th Amendment requirement that D

be "informed of the nature and cause of the accusation" in criminal cases. However, we believe that changing the order is practically possible, in part because civil law criminal cases usually start with interrogation of the defendant (cf. Damaska (1973) p528). On the other hand, we have also shown that litigants who always share available evidence cannot prefer to choose an order after observing the available evidence.

We suggested in the Introduction that our results may shed light on debates in which participants cannot lie (e.g. because there are fact-checkers), but which lack some of the procedural rules of common law trials. We end the paper by discussing the robustness of our results on discovery games to alternative rules. We first show that a litigant may be advantaged by the presence of a litigant with opposite preferences (Section 8.1). We then explore the implications of alternative stopping rules (Section 8.2), of multiple possible verdicts (Section 8.3) and of Full Reports (Section 8.4).

8.1. Advantageous competition

We have focused throughout on two-litigant games; but a comparison with one-litigant games is instructive. Specifically, consider a game (say, Γ^D) in which a single litigant (say, D) presents evidence to J, which then reaches a verdict. D might not present evidence which directly proves guilt in equilibrium; so it is not surprising that Γ^D may not have a separating equilibrium. Example 8 below illustrates how the availability of evidence which directly proves guilt may result in a wrongful *conviction*:

Example 8 *There are three states: $S = \{1, 2, c\}$, and the defendant is only factually guilty in state c. There are three witnesses, $e = \{1, c\}$, $f = \{2\}$ and $g = \{c\}$, and three evidence sets, $E = \{e\}$, $F = \{f\}$ and $G = \{e, g\}$, whose conditional distribution satisfies $\pi_1(E) = \pi_2(F) = \pi_c(G) = 1$. The prior distribution satisfies $\frac{p_c}{p_c+p_1} > d$.*■

D cannot present witness g in any equilibrium of Γ^D because J would then acquit after observing e; so D could profitably deviate to presenting e at G. The condition on priors implies that J must convict at E and at G in every equilibrium of Γ^D.

D's problem in Γ^D is a commitment failure. Specifically, consider the commitment game Γ^D_c in which D chooses the evidence it would present at each evidence set before observing Nature's choice. D would commit to presenting witness g at G in every equilibrium of Γ^D; and J would then acquit at E.

Now consider the discovery game $\Gamma^{D,P}$ which satisfies the conditions in Example 8. This game has two outcomes. One equilibrium replicates the outcome of Γ^D: D presents e at E and at G, to which P responds by passing. Proposition 1 implies that $\Gamma^{D,P}$ also has a separating equilibrium: for example, D presents and P completes the full report at every evidence set. J acquits at E in these equilibria. Applying our criteria: both D and J ex ante prefer to play $\Gamma^{D,P}$ over playing Γ^D.[34] Perhaps yet more strikingly, Γ^P has the same equilibrium outcomes as $\Gamma^{D,P}$: so D would ex ante prefer to delegate presentation of evidence to P.

[34] Theorem 1a) implies that D and J also ex ante prefer to play $\Gamma^{P,D}$ over playing Γ^D.

8.2. Stopping rules

We have obtained our results in a model where litigants only have one opportunity each to present witnesses. This assumption precludes the presentation of rebuttal evidence; but such evidence is arguably redundant in our model, on the interpretation that interrogation and cross-examination reveal exactly what each witness knows. In any case, common law courts do not allow the leader to present new evidence after the follower has presented. Nevertheless, it is instructive to consider equilibrium play in games with alternative stopping rules, which may be more appropriate to debates other than trials.

First, consider a debate between two litigants who alternately present evidence or pass over any fixed number of rounds, as in Lipman and Seppi (1995):[35] a set-up akin to Presidential debates. If litigants share available evidence then the game has an equilibrium in which evidence is only presented in the last two rounds - which may shed light on Obama's success in the 2012 debate with Romney.

Alternatively, consider games which only differ from our model in the following respect: litigants alternate in presenting evidence (starting in Round 1) until a litigant passes, at which point J reaches a verdict and the game ends. We can show that Proposition 1 and Lemma hold in discovery games;[36] so no litigant can prefer to present in odd-numbered rounds. However, litigants cannot prefer to present in even-numbered rounds. To see this, recall our analysis of Example 2, which we used to prove that litigants in our model can prefer to present second:

We argued in Section 4 that our version of Example 2 has an equilibrium in which J observes $e, pass$ and acquits at E and at H if P presents first. It is easy to confirm that this is also an equilibrium outcome with the alternative stopping rule if P presents in odd-numbered rounds. We also argued that every equilibrium must be separating in our model if D presents first because D could profitably deviate to presenting g at E unless J acquits after observing e, f - in which case, D could profitably deviate to presenting e at F. This argument fails with the alternative stopping rule because D does not forego the opportunity to directly prove innocence at E by presenting e in Round 1. Specifically, there is an equilibrium in which J convicts after observing $e, f, pass$ and acquits after observing $e, f, g, pass$.[37] D can no longer profitably deviate to presenting e at F in Round 1, while P cannot profitably deviate from passing to presenting f at E. This argument implies that the same non-separating outcome exists, irrespective of the order of presentation. We draw the following lesson. In our model, the leader commits to the evidence which it decides to present; with the alternative stopping rule, a litigant who does not call some witness in one round can do so later if its rival has not done so.

These issues could be posed more generally. In some debates, an impatient J can commit to the stopping rule. It would be interesting to know whether the order matters in an optimal mechanism. If J could design debate rules then it would surely require the analog of closing statements if its memory were imperfect. Absent such procedural rules, the memory and anchoring effects which psychologists highlight would presumably have

[35] Strictly speaking, Lipman and Seppi do not require litigants to alternate.

[36] A proof is available from the authors on request.

[37] $e, f, pass$ means that D presents e in Round 1 and passes in Round 3; $e, f, g, pass$ should be read analogously.

greater consequence than in trials.

8.3. Multiple verdicts

According to our model of trials, J reaches a binary verdict, either acquitting or convicting the defendant. In some civil trials, J must decide on the level of damages; and the defendant faces a number of charges in some criminal trials. It is therefore interesting to consider whether our results carry over to games in which J can reach multiple verdicts. Suppose, in particular, that the set of possible verdicts is an interval, and that J has smooth, strictly concave preferences over verdicts in this interval whenever it knows the evidence set pair. D [resp. P] then seeks to minimize [resp. maximize] the verdict. Consider games of this sort played under discovery.

The construction used above to prove Proposition 1 can be amended to show that games with a continuum of possible verdicts possess a separating equilibrium. A litigant can therefore only prefer an order if it uniformly prefers an outcome of a game over the separating outcome. Our assumptions about J's preferences preclude this possibility: J can only reach a non-separating verdict $v \neq v(E)$ at E if litigants present the same evidence pair at E and at another evidence set F such that $v(E) \neq v(F)$. The verdict at E must then be a compromise between $v(E)$ and $v(F)$; so a litigant who strictly prefers v over $v(E)$ must strictly prefer the separating verdict at another evidence set over v. In sum, litigants cannot prefer an order when there is a continuum of possible verdicts. Note, though, that this argument does not preclude exactly one of the games possessing a unique outcome; so J could prefer an order in this set-up.

The argument in the last paragraph does not apply with a finite number of verdicts. While the construction used to prove Lemma cannot be extended to this case, we conjecture that Lemma still holds. We can also construct examples in which litigants prefer to present second. Accordingly, we conjecture that Theorem 1 carries over to games with any finite number of verdicts.

8.4. Full reports

We argued above that Full Reports is a natural assumption when modelling common law trials; but Full Reports might fail in other debates, e.g. because the listener has a limited attention span. Discovery games which fail Full Reports may lack a separating equilibrium: for example, if litigants could only present one witness and the conditions in Example 1 held (cf. Section 3.3) then $\Gamma^{D,P}$ would have a separating equilibrium because P could induce conviction by presenting whichever witness D did not present. However, $\Gamma^{P,D}$ would not have a separating equilibrium because D would then always pass at MO. However, both of these games have an equilibrium in which J observes $o, pass$ (and then convicts) at O and at MO. Consequently, litigants prefer to present first in this variant on a discovery game.

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